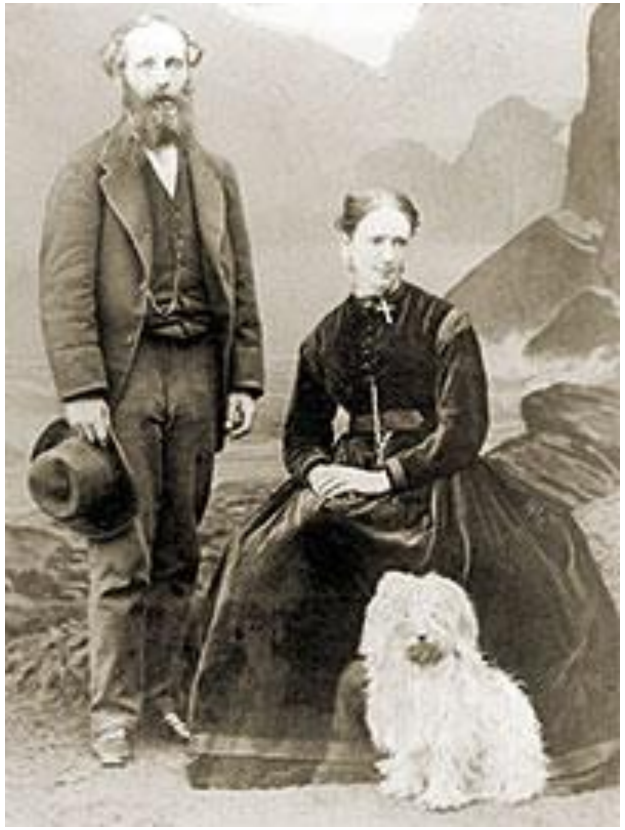
JAMES CLERK MAXWELL:

LAS CUATRO ECUACIONES QUE CAMBIARON EL MUNDO

Día Internacional de la Luz

16 de mayo

UNIVERSIDAD DE CANTABRIA



Santander, 9 de Mayo de 2018

Maxwell era un hombre discreto (pero muy preguntón y curioso). Sus biógrafos dicen que si se hace una encuesta, la gente normalmente no nombra nadie entre Newton y Einstein

“Una época científica acabó y otra empezó con JCM”

“El trabajo de James Clerk Maxwell cambió el mundo para siempre”

“Si he logrado ver más lejos es porque he subido a hombros de Gigantes” escribió **Isaac Newton** a su rival Robert Hooke en 1676.

250 años después, durante una de las visitas que **Albert Einstein** realizó a Cambridge (Inglaterra), alguien le señaló que él había llegado tan lejos porque se había subido a hombros de Newton.

Einstein le replicó tajante: **“Eso no es cierto, estoy subido a hombros de Maxwell”**

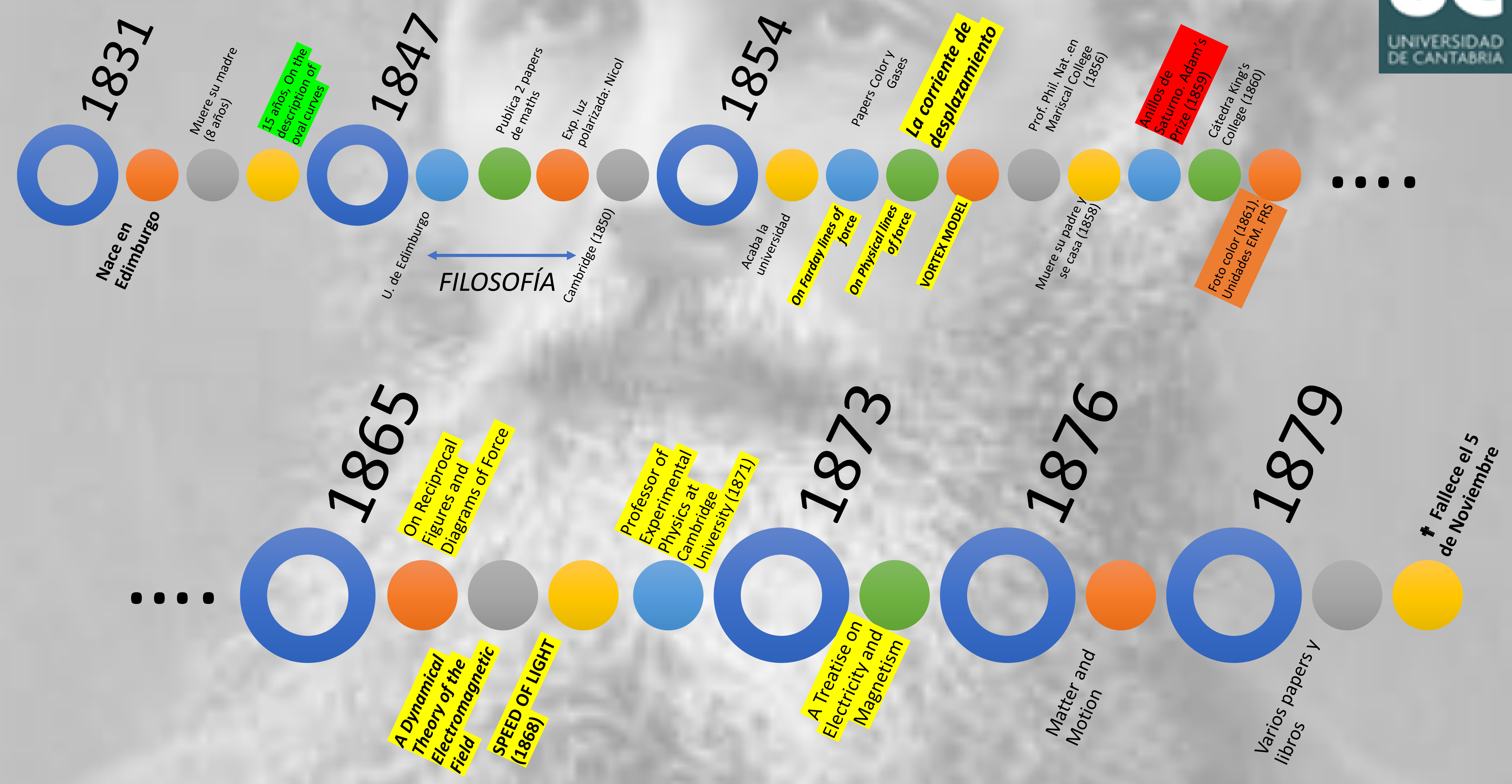
17 Equations That Changed the World
by Ian Stewart

NEWTON

MAXWELL

EINSTEIN

1.	**Pythagoras's Theorem**	$a^2+b^2=c^2$	Pythagoras,530 BC
2.	**Logarithms**	$\log xy = \log x + \log y$	John Napier, 1610
3.	**Calculus**	$\frac{\mathrm{d}f}{\mathrm{d}t} = \lim_{h\to 0} = \frac{f(t+h)-f(t)}{h}$	Newton, 1668
4.	**Law of Gravity**	$F = G\frac{m_1 m_2}{r^2}$	Newton, 1687
5.	**The Square Root of Minus One**	$i^2=-1$	Euler, 1750
6.	**Euler's Formula for Polyhedra**	$V-E+F=2$	Euler, 1751
7.	**Normal Distribution**	$\Phi(x) = \frac{1}{\sqrt{2\pi\rho}}e^{\frac{(x-\mu)^2}{2\rho^2}}$	C.F. Gauss, 1810
8.	**Wave Equation**	$\frac{\partial^2 u}{\partial t^2} = c^2\frac{\partial^2 u}{\partial x^2}$	J. d'Almbert, 1746
9.	**Fourier Transform**	$f(\omega) = \int_{\infty}^{\infty} f(x)e^{-2\pi i x\omega}\mathrm{d}x$	J. Fourier, 1822
10.	**Navier-Stokes Equation**	$\rho\left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v}\cdot\nabla\mathbf{v}\right) = -\nabla p + \nabla\cdot\mathbf{T} + \mathbf{f}$	C. Navier, G. Stokes, 1845
11.	**Maxwell's Equations**	$\nabla\cdot\mathbf{E}=0$ $\nabla\cdot\mathbf{H}=0$ $\nabla\times\mathbf{E} = -\frac{1}{c}\frac{\partial\mathbf{H}}{\partial t}$ $\nabla\times\mathbf{H} = \frac{1}{c}\frac{\partial E}{\partial t}$	J.C. Maxwell, 1865
12.	**Second Law of Thermodynamics**	$\mathrm{d}S \geq 0$	L. Boltzmann, 1874
13.	**Relativity**	$E=mc^2$	Einstein, 1905
14.	**Schrodinger's Equation**	$i\hbar\frac{\partial}{\partial t}\Psi = H\Psi$	E. Schrodinger, 1927
15.	**Information Theory**	$H = -\sum p(x)\log p(x)$	C. Shannon, 1949
16.	**Chaos Theory**	$x_{t+1} = kx_t(1-x_t)$	Robert May, 1975
17.	**Black-Scholes Equation**	$\frac{1}{2}\sigma^2 S^2\frac{\partial^2 V}{\partial S^2} + rS\frac{\partial V}{\partial S} + \frac{\partial V}{\partial t} - rV = 0$	F. Black, M. Scholes, 1990



UC
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EL ENTORNO CIENTÍFICO ANTERIOR (JCM nace en 1831)

1785

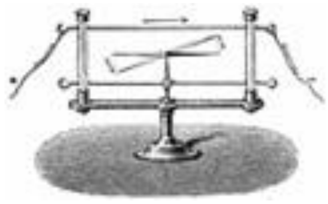
Charles-Augustin de Coulomb AFIRMA QUE LA FUERZA ENTRE DOS CARGAS VARÍA CON LA INVERSA DEL CUADRADO DE LA DISTANCIA

1800

Alessandro Volta INVENTA LA BATERÍA QUE PERMITE A LOS EXPERIMENTALES TRABAJAR CON CORRIENTES CONTINUAS

1820

Hans Christian Ørsted LA AGUJA DE UNA BRÚJULA SE MUEVE CUANDO SE APROXIMA A UN HILO QUE TRANSPORTA CORRIENTE. PRIMERA EVIDENCIA ENTRE ELECTRICIDAD Y MAGNETISMO



André-Marie Ampère DOS HILOS DE CORRIENTE SE ATRAEN O SE REPELEN ”MOLÉCULA ELECTRODINÁMICA”→ACTUAL ELECTRÓN

Maxwell y la filosofía: de los 16 a los 19

- ✓ U. Edimburgo → Filosofía Natural, Matemáticas y Lógica (Sir William Hamilton):

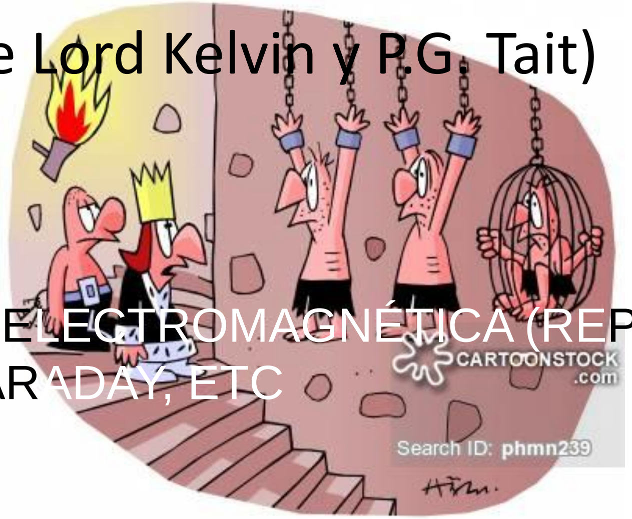
  Now the only thing which can be directly perceived by the senses is Force, to which may be reduced light, heat, electricity, sound and all the other things which can be perceived by the senses.

- ✓ Influencia de James Forbes en el aspecto filosófico y en el experimental (decía: *nunca disuado a nadie de hacer un experimento. Si no encuentra lo que busca, encuentra algo diferente*). Le corrige la manera de escribir. Se inspira en Forbes desde joven y en un review de Nature escribió años más tarde:

  If a child has any latent talent for the study of nature, a visit to a real man of science at work in his laboratory may be a turning point in his life. He may not understand a word of what the man of science says to explain his operations; but he sees the operations themselves, and the pains and patience which are bestowed on them; and when they fail he sees how the man of science, instead of getting angry, searches for the cause of failure among the conditions of the operation.

- ✓ Newton (Optics), Fourier (250 aniversario), Cauchy (cálculo diferencial), Poisson (mecánica). Hace pequeños experimentos electromagnéticos. Juega con la **polarización de la luz** (¡*sin polarizadores*!). Nicol le regala un prisma
- ✓ Escribe tres trabajos de matemáticas.

QUIZÁ, EL MÁS INFLUYENTE (Aparte de Lord Kelvin y P.G. Tait)



"Which one of you is called Faraday?"

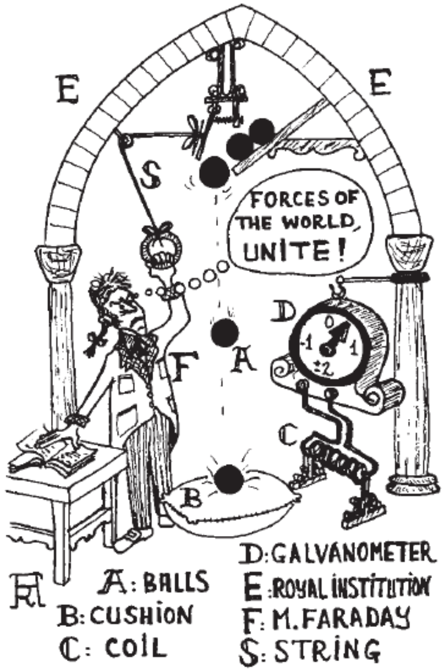
1831

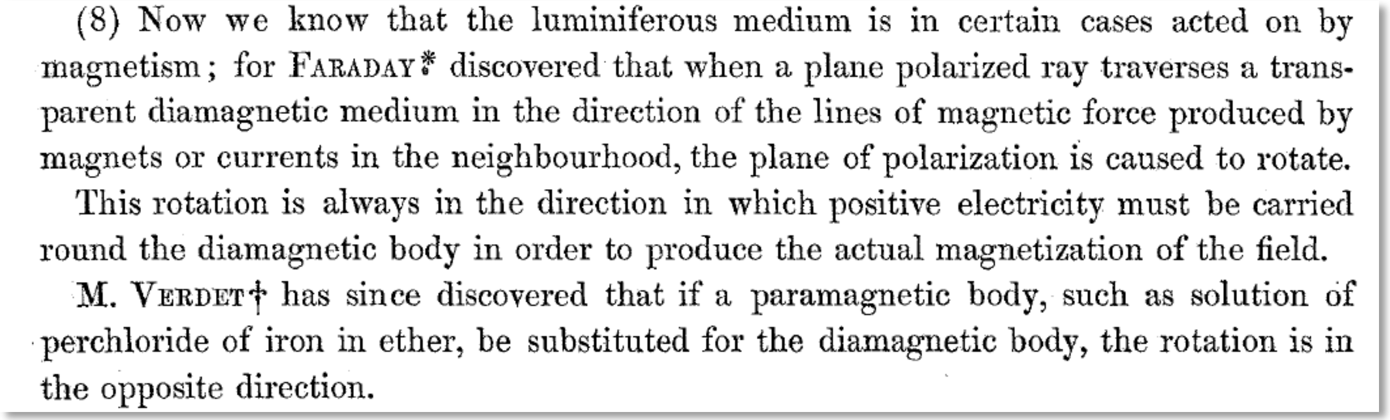
Michael Faraday (1791-1867), INDUCCION ELECTROMAGNÉTICA (REPITE EL EXPERIMENTO DE ØRSTED) , EFECTO FARADAY, ETC

CONCEPTO DE **ACCION A DISTANCIA** (NEWTON)

- FARADAY→LINEAS DE FUERZA
- MAXWELL→CALOR→CONCEPTO DE CAMPO (FLUJO)

LA UNIFICACIÓN DE LAS FUERZAS





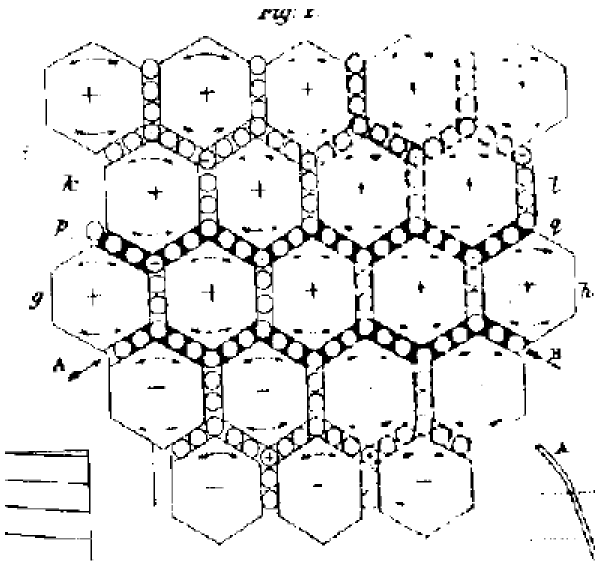
(8) Now we know that the luminiferous medium is in certain cases acted on by magnetism; for FARADAY* discovered that when a plane polarized ray traverses a transparent diamagnetic medium in the direction of the lines of magnetic force produced by magnets or currents in the neighbourhood, the plane of polarization is caused to rotate. This rotation is always in the direction in which positive electricity must be carried round the diamagnetic body in order to produce the actual magnetization of the field. M. VERDET† has since discovered that if a paramagnetic body, such as solution of perchloride of iron in ether, be substituted for the diamagnetic body, the rotation is in the opposite direction.

Citado 6 veces y mencionado 3 en J. C. Maxwell *Phil. Trans. R. Soc. Lond.* **155**, 459-512 (1865)

LA SOMBRA DE NEWTON: LA CORRIENTE DE DESPLAZAMIENTO

We have therefore some reason to believe, from the phenomena of light and heat, that there is an æthereal medium filling space and permeating bodies, capable of being set in motion and of transmitting that motion from one part to another, and of communicating that motion to gross matter so as to heat it and affect it in various ways.

Prop. XIV.—To correct the equations (9) † of electric currents for the effect due to the elasticity of the medium.

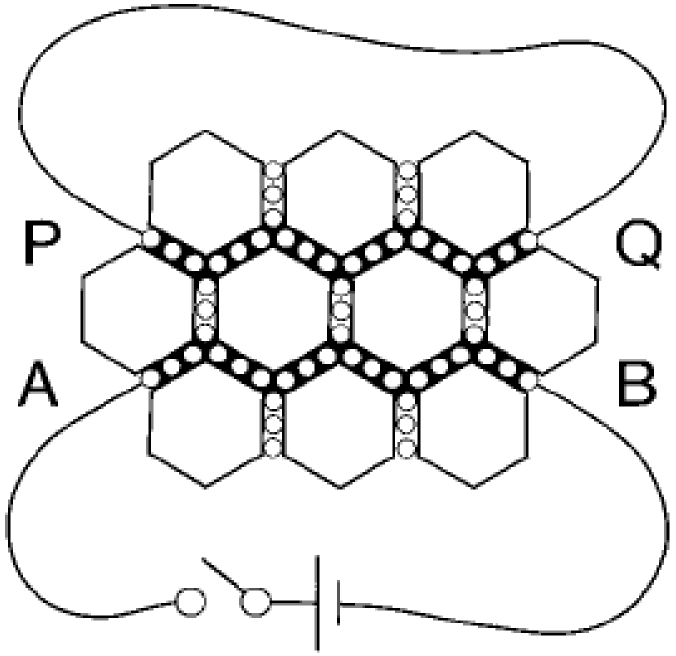


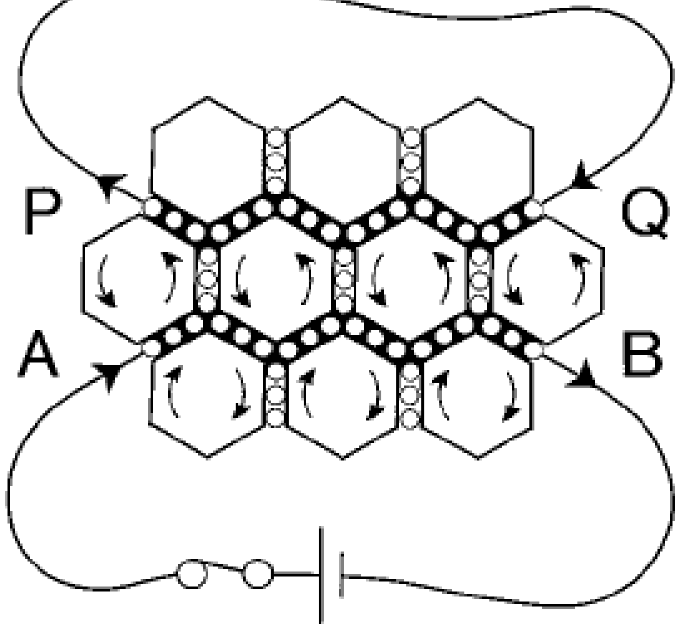
The Molecular Vortex Model

- Materiales magnéticos y no magnéticos→células más o menos densas
- Materiales conductores o dieléctricos→bolitas más o menos móviles

J. C. Maxwell, "*On Faraday's Lines of Force*," Transactions of the Cambridge Philosophical Society, **10 Part I**, 1864, pp. 27-65 (read December 10, 1855, and February 11, 1856)

UN EJEMPLO DE ESTA "MECÁNICA"





FORMA DE ECUACIONES ANTIGUAS

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Equations		No.	Law
$e+\frac{df}{dx}+\frac{dg}{dy}+\frac{dh}{dz}=0$	G	(1)	Gauss' Law
$\mu\alpha=\frac{dH}{dy}-\frac{dG}{dz}$ $\mu\beta=\frac{dF}{dz}-\frac{dH}{dx}$ $\mu\gamma=\frac{dG}{dx}-\frac{dF}{dy}$	B	(2)	Equivalent to Gauss' Law for magnetism
$P=\mu\left(\gamma\frac{dy}{dt}-\beta\frac{dz}{dt}\right)-\frac{dF}{dt}-\frac{d\Psi}{dx}$ $Q=\mu\left(\alpha\frac{dz}{dt}-\gamma\frac{dx}{dt}\right)-\frac{dG}{dt}-\frac{d\Psi}{dy}$ $R=\mu\left(\beta\frac{dx}{dt}-\alpha\frac{dy}{dt}\right)-\frac{dH}{dt}-\frac{d\Psi}{dz}$	D	(3)	Faraday's Law (with the Lorentz Force and Poisson's Law)
$\frac{d\gamma}{dy}-\frac{d\beta}{dz}=4\pi p'$ $p'=p+\frac{df}{dt}$ $\frac{d\alpha}{dz}-\frac{d\gamma}{dx}=4\pi q'$ $q'=q+\frac{dg}{dt}$ $\frac{d\beta}{dx}-\frac{d\alpha}{dy}=4\pi r'$ $r'=r+\frac{dh}{dt}$	C	(4)	Ampère-Maxwell Law
$P=-\xi p \quad Q=-\xi q \quad R=-\xi r$	F		Ohm's Law
$P=kf \quad Q=kg \quad R=kh$	E		The electric elasticity equation ($\mathbf{E}=\mathbf{D}/\varepsilon$)
$\frac{de}{dt}+\frac{dp}{dx}+\frac{dq}{dy}+\frac{dr}{dz}=0$	H		Continuity of charge

For Electromagnetic Momentum	F	G	H
,, Magnetic Intensity	α	β	γ
,, Electromotive Force	P	Q	R
,, Current due to true conduction	p	q	r
,, Electric Displacement	f	g	h
,, Total Current (including variation of displacement)	p'	q'	r'
,, Quantity of free Electricity	e		
,, Electric Potential	Ψ		

Between these twenty quantities we have found twenty equations, viz.

Three equations of Magnetic Force	(B)
,, Electric Currents	(C)
,, Electromotive Force	(D)
,, Electric Elasticity	(E)
,, Electric Resistance	(F)
,, Total Currents	(A)
One equation of Free Electricity	(G)
,, Continuity	(H)

These equations are therefore sufficient to determine all the quantities which occur in them, provided we know the conditions of the problem. In many questions, however, only a few of the equations are required.

$\mathbf{J}_{total}=\mathbf{J}_{conduction}+\partial\mathbf{D}/\partial t$	(A)
$\mathrm{curl}\ \mathbf{A}=\mu\mathbf{H}$	(B)
$\mathrm{curl}\ \mathbf{H}=\mathbf{J}_{total}$	(C)
$\mathbf{E}=\mu\mathbf{v}\times\mathbf{H}-\partial\mathbf{A}/\partial t-\mathrm{grad}\psi$	(D)
$\mathbf{D}=\varepsilon\mathbf{E}$	(E)
$\mathbf{E}=R\mathbf{J}_{conduction}$	(F)
$\mathrm{div}\ \mathbf{D}=\rho$	(G)
$\mathrm{div}\ \mathbf{J}+\partial\rho/\partial t=0$	(H)

FORMALISMO LAGRANGIANO→FORMALISMO HAMILTONIANO

LAS ECUACIONES MODERNAS Y CÓMO JCM LAS INTERPRETABA

$$\vec{\nabla}\cdot\vec{B}=0$$

$$\vec{\nabla}\times\vec{B}=\mu\left(\vec{J}+\varepsilon\frac{\partial\vec{E}}{\partial t}\right)$$

$$\vec{\nabla}\times\vec{E}=-\frac{\partial\vec{B}}{\partial t}$$

$$\vec{\nabla}\cdot\vec{E}=\frac{\rho}{\varepsilon}$$

These equations express—

(A) The relation between electric displacement, true conduction, and the total current, compounded of both.

(B) The relation between the lines of magnetic force and the inductive coefficients of a circuit, as already deduced from the laws of induction.

(C) The relation between the strength of a current and its magnetic effects, according to the electromagnetic system of measurement.

(D) The value of the electromotive force in a body, as arising from the motion of the body in the field, the alteration of the field itself, and the variation of electric potential from one part of the field to another.

(E) The relation between electric displacement, and the electromotive force which produces it.

(F) The relation between an electric current, and the electromotive force which produces it.

(G) The relation between the amount of free electricity at any point, and the electric displacements in the neighbourhood.

(H) The relation between the increase or diminution of free electricity and the electric currents in the neighbourhood.

There are twenty of these equations in all, involving twenty variable quantities.

(20) The general equations are next applied to the case of a magnetic disturbance propagated through a non-conducting field, and it is shown that the only disturbances which can be so propagated are those which are transverse to the direction of propagation, and that the velocity of propagation is the velocity v, found from experiments such

This velocity is so nearly that of light, that it seems we have strong reason to conclude that light itself (including radiant heat, and other radiations if any) is an electromagnetic disturbance in the form of waves propagated through the electromagnetic field according to electromagnetic laws. If so, the agreement between the elasticity of the

The conception of the propagation of transverse magnetic disturbances to the exclusion of normal ones is distinctly set forth by Professor FARADAY* in his "Thoughts on Ray Vibrations." The electromagnetic theory of light, as proposed by him, is the same in substance as that which I have begun to develope in this paper, except that in 1846 there were no data to calculate the velocity of propagation.

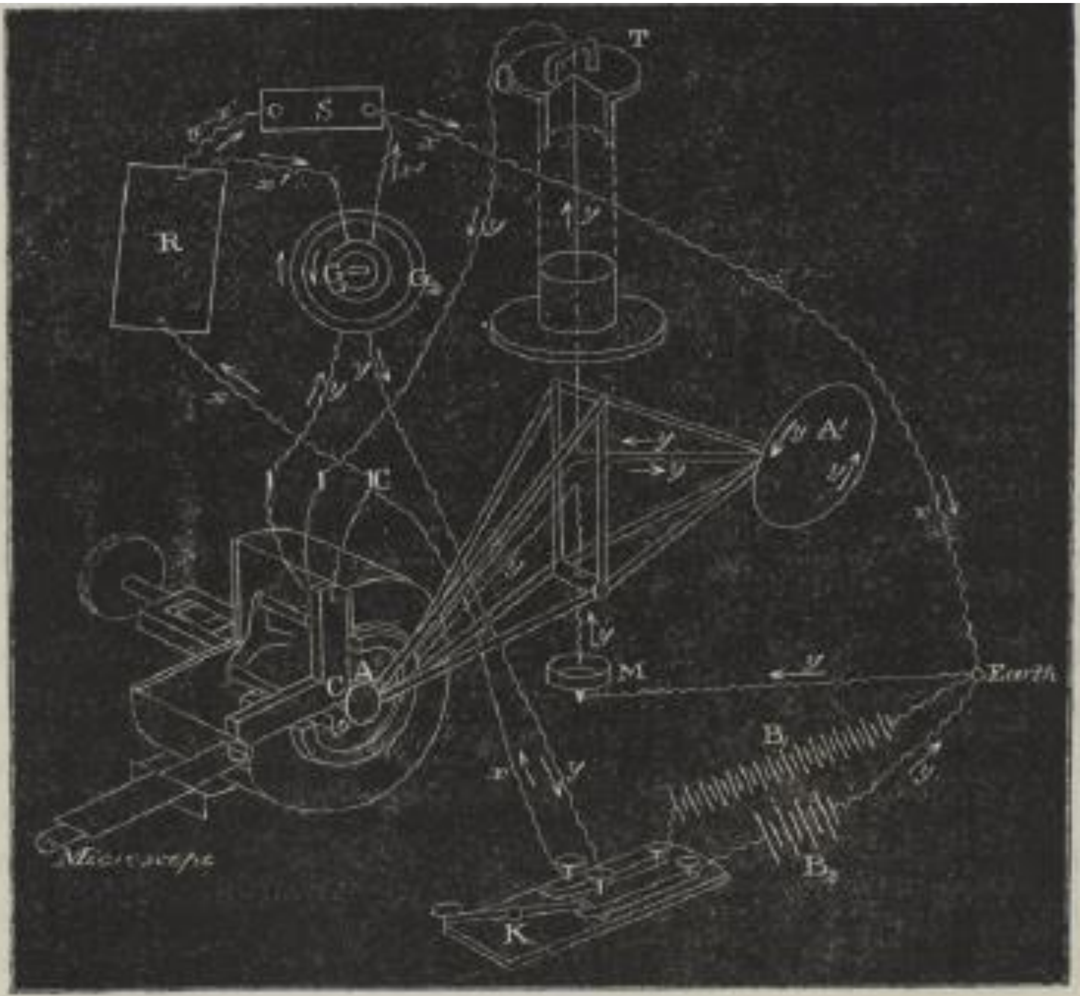
J. Clerk Maxwell, Phil. Trans. R. Soc. Lond. 1865 155, 459-512, published 1 January 1865

XXVI. *On a Method of making a Direct Comparison of Electrostatic with Electromagnetic Force; with a Note on the Electromagnetic Theory of Light.* By J. CLERK MAXWELL, *F.R.SS. L. & E.*

Velocity of light (km/s)		Ratio of electrical units (km/s)	
Fizeau	314,000	Weber	310,740
Aberration etc. and sun's parallax	308,000	Maxwell	288,000
Fouceau	298,360	Thomson	282,000

Valor exacto = 299 792 458 m/s

Phil. Trans. R. Soc. Lond. 1868 158, 643-657, published 1 January 1868



A. Suspended disk and coil.
A′. Counterpoise disk and coil.
C. Fixed disk and coil.
B_1. Great battery. B_2. Small battery.
G_1. Primary coil of galvanometer. G_2. Secondary coil.
R. Great resistance. S. Shunt.

K. Double key. *g*. Graduated glass scale.
C. Electrode of fixed disk.
x. Current through R.
x′. Current through G_1. $x-x'$. Current through S.
y. Current through the three coils and G_2.
M. Mercury cup. T. Torsion head and tangent screw.

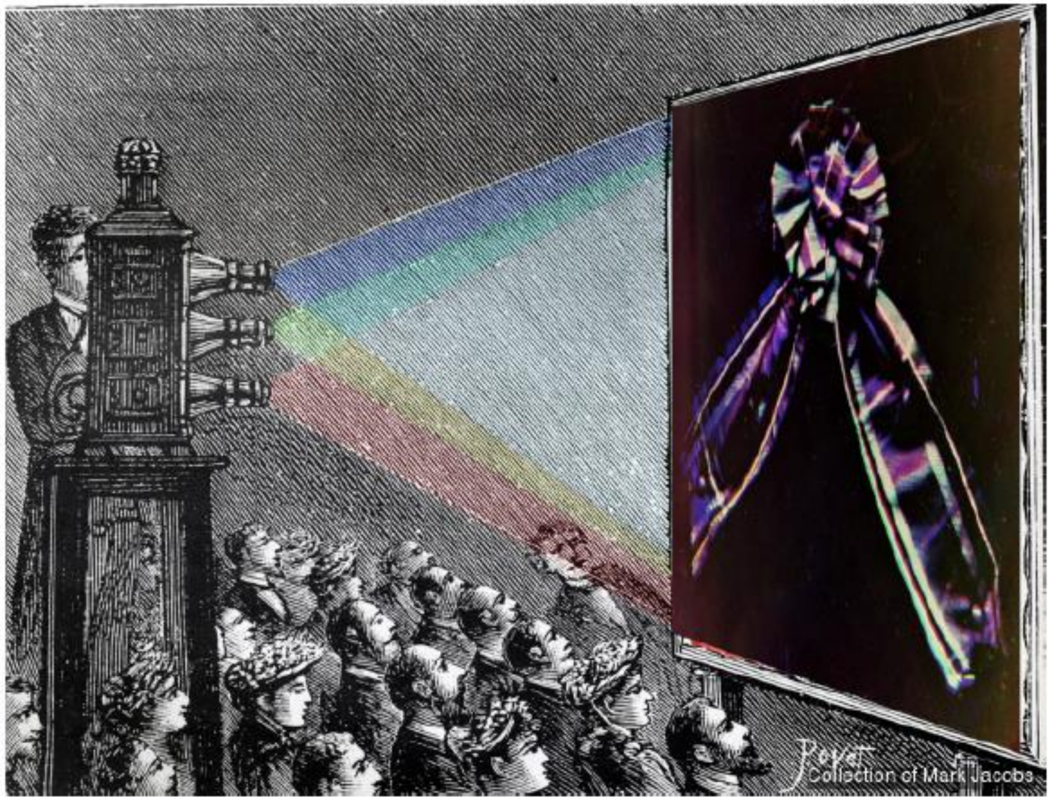
One quarter of the micrometer-box, disks, and coils is cut away to show the interior. The case of the instrument is not shown. The galvanometer and shunts were 10 feet from the Electric Balance.

LA GRAN APORTACIÓN AL COLOR de JCM

('but how d'ye know it's blue?')

Violet, Indigo, Blue, Green, Yellow, Orange and Red

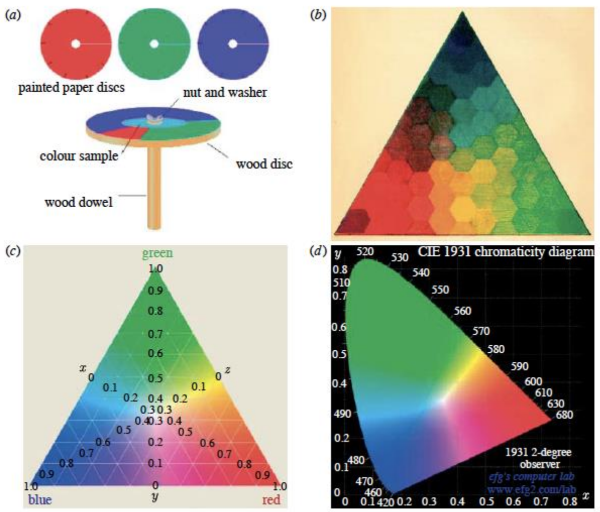
La historia comienza en Newton (1670) con su "experimentum crucis" (VIBGYOR Spectrum) y su telescopio reflector, continúa con Young con la teoría aditiva del color (1802) y JCM la rubrica: XVIII.—**Experiments on Colour, as perceived by the Eye, with Remarks on Colour-Blindness**. *Transactions of the Royal Society of Edinburgh, 21*(2), 275-298 (1857).

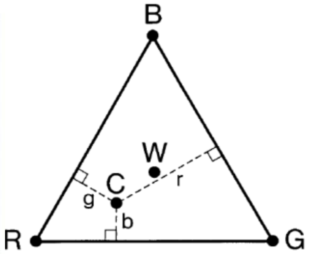


Rumford Medal de la Royal Society of London

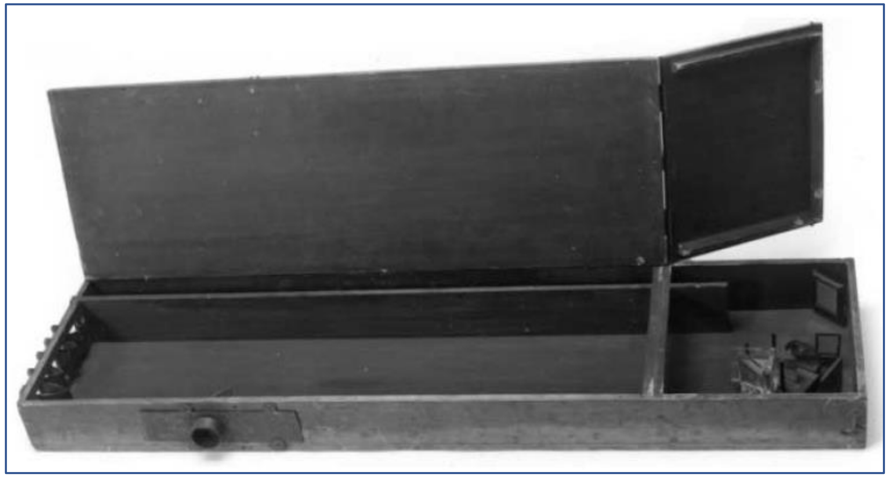
The colours used for Mr Purdie's papers were—

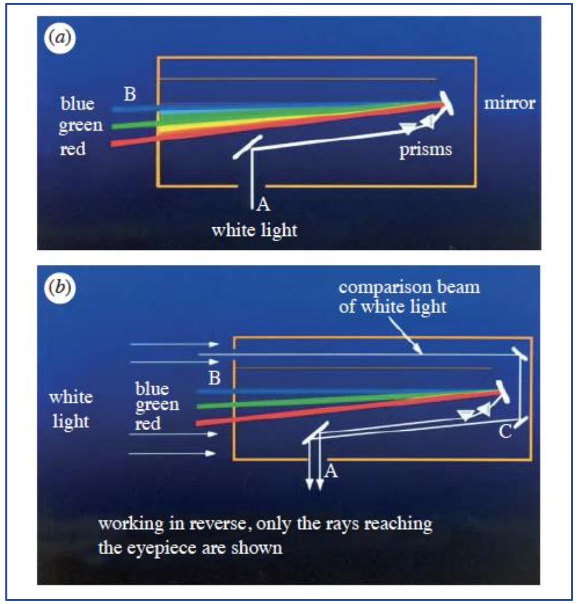
Vermilion,	V	Ultramarine,	U	Emerald Green,	EG
Carmine,	C	Prussian Blue,	PB	Brunswick Green,	BG
Red Lead,	RL	Verditer Blue,	VB	Mixture of Ultramarine and Chrome,	UC
Orange Orpiment,	OO				
Orange Chrome,	OC				
Chrome Yellow,	CY				
Gamboge,	Gam				
Pale Chrome,	PC				
		Ivory Black,	Bk		
		Snow White,	SW		
		White Paper (Pirie, Aberdeen).			





Más de 200 observaciones





Maxwell, J. (1857). XVIII.—Experiments on Colour, as perceived by the Eye, with Remarks on Colour-Blindness. *Transactions of the Royal Society of Edinburgh, 21*(2), 275-298.

Phil. Trans. R. Soc. A (2008) 366, 1685–1696

UN PARÉNTESIS

Displacement current:

$$J_D = \varepsilon_0 \frac{\partial D}{\partial t}$$

$$J_D = \varepsilon_0 \frac{\partial E}{\partial t} + \frac{\partial P}{\partial t}$$

Fundamentals:

- Electromagnetic wave;
- Electromagnetic induction;
- Theory for light.

Technology impacts (1886-):

- Antenna;
- Telegram, Radio, TV;
- Radar; microwave;
- Wireless communication.

Fundamentals:

- Polarization induced current.

Technology impacts:

- Nanogenerators (PENG, TENG) for energy (2006-);
- Self-powered sensors and systems (2006-);
- Electronic components (capacitors).

Impacts (2010-):

- Internet of things;
- Sensor network;
- Blue energy;
- Big data

LA CORREINTE DE DESPLAZAMIENTO "MODERNA"

Zhong Lin Wang, Materials Today **20** (2) 74-82 (2017)

EL ENTORNO CIENTÍFICO POSTERIOR (✝1879)

UNIVERSIDAD DE CANTABRIA

1878

Los Físicos **George Francis FitzGerald** and **Oliver Lodge** TRABAJAN JUNTOS PARA DEMOSTRAR QUE LA LUZ ES UNA OEM

1881

Los Físicos **Albert Abraham Michelson** (Premio Nobel de Física, 1907) y **Edward Morley** REALIZAN EL EXPERIMENTO QUE TIRA POR TIERRA LA TEORÍA DEL ETER.

1885

Oliver Heaviside PUBLICA UNA VERSIÓN CONDENSADA DE LAS ECUACIONES, REDUCIÉNDOLAS A 4 EN VEZ DE 20. "*I never made any progress until I threw all the potentials overboard*". LA NUEVA FORMULACIÓN CONVERTÍA AL CAMPO ELÉCTRICO Y AL MAGNÉTICO EN PROTAGONISTAS.

1888

Heinrich Hertz, EN UN LABORATORIO BIEN EQUIPADO EN KARSLRUHE, CONFIRMA LA EXISTENCIA DE LAS ONDAS ELECTROMAGNÉTICAS PREDICHA POR MAXWELL.

Los tiempos del electromagnetismo

~100 años

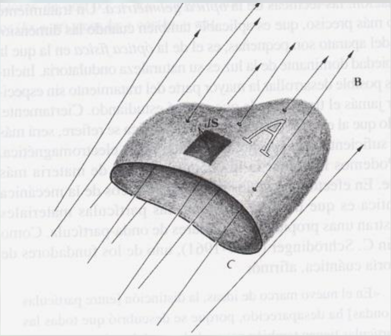
- 1785, Se publica la ley de Coulomb
- 1812, Se publica la ley de Poisson
- 1813, Se descubre el Teorema de la Divergencia de Gauss
- 1820, H.C. Ørsted descubre que una corriente eléctrica genera un campo magnético
- 1820, André-Marie Ampère --> electrodinámica; se descubre la ley de Biot-Savart
- 1826, Se publica la ley de Ampère
- 1831, Se publica la ley de Faraday
- 1856, James Clerk Maxwell publica "On Faraday's lines of force"
- 1861, Maxwell publica "On physical lines of force"
- 1865, Maxwell publica "A dynamical theory of the electromagnetic field"
- 1873, Maxwell publica "Treatise on Electricity and Magnetism"
- 1888, Heinrich Hertz descubre las ondas de radio
- 1940, Albert Einstein populariza el nombre de "Ecuaciones de Maxwell"
- 1966, Kane Yee introduce el método FDTD para resolver las ecuaciones de Maxwell

< 1/3

Ecuaciones de Maxwell.- Origen y formulación integral (S.I.)

• **Ley de inducción de Faraday**

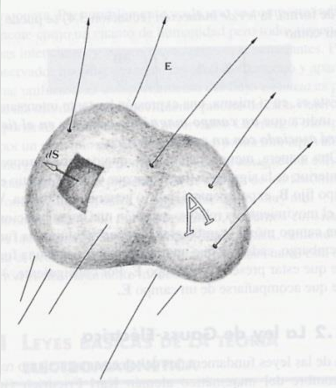
$$\oint_C \vec{E}\cdot\vec{dl} = -\iint_A \frac{\partial \vec{B}}{\partial t}\, d\vec{S}$$



• **Ley de Gauss para el campo eléctrico**

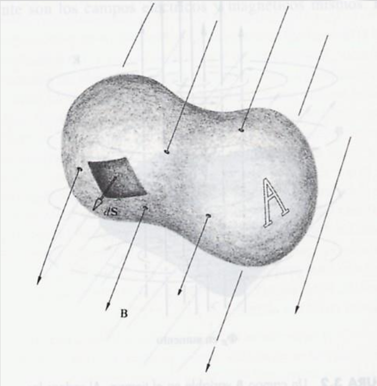
$$\oiint_A \vec{E}\cdot\vec{dS} = \frac{1}{\varepsilon}\iiint_V \rho\cdot dV$$

— (ρ) —



• **Ley de Gauss para el campo magnético**

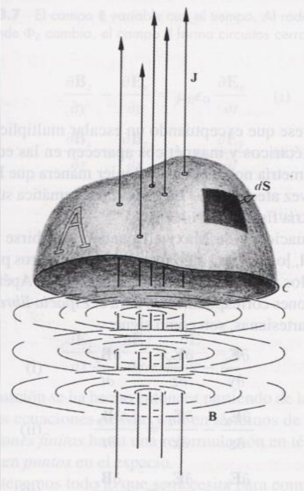
$$\oiint_A \vec{B}\cdot\vec{dS} = 0$$



• **Ley circuital de Ampère-Maxwell**

$$\oint_C \vec{B}\cdot\vec{dl} = \mu\iint_A \left[\vec{j} + \varepsilon\frac{\partial \vec{E}}{\partial t}\right]\vec{dS}$$

— (J) —



Ecuaciones de Maxwell.- Formulación diferencial

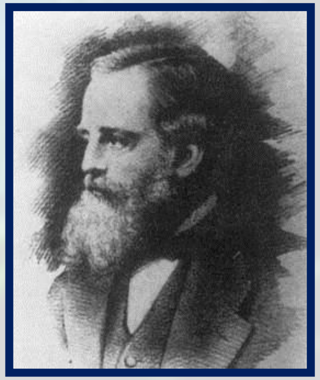
Maxwell's vector equations taught in university are actually Heaviside's truncated equations.

$$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$

$$\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\varepsilon}$$

$$\vec{\nabla}\cdot\vec{B} = 0$$

$$\vec{\nabla}\times\vec{B} = \mu\left[\vec{J} + \varepsilon\frac{\partial\vec{E}}{\partial t}\right]$$



$\vec{E}$

Se observa gran simetría. Heaviside: *añade carga magnética*

$\vec{E}$ $\vec{B}$ acoplados

$$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} \quad \textbf{(1)}$$

$$\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\varepsilon} \quad \textbf{(2)}$$

$$\vec{\nabla}\cdot\vec{B} = 0 \quad \textbf{(3)}$$

$$\vec{\nabla}\times\vec{B} = \mu\sigma\vec{E} + \mu\varepsilon\frac{\partial\vec{E}}{\partial t} \quad \textbf{(4)}$$

Tomo el rotacional de (4) $\vec{\nabla}\times\left(\vec{\nabla}\times\vec{B}\right) = \mu\sigma\left(\vec{\nabla}\times\vec{E}\right) + \mu\varepsilon\frac{\partial}{\partial t}\left(\vec{\nabla}\times\vec{E}\right)$

Sustituyo (1) y obtengo $\vec{\nabla}\times\left(\vec{\nabla}\times\vec{B}\right) = -\mu\sigma\frac{\partial\vec{B}}{\partial t} - \mu\varepsilon\frac{\partial^2}{\partial t^2}\left(\vec{B}\right)$

Desarrollo el 1er miembro: $\vec{\nabla}\times\left(\vec{\nabla}\times\vec{B}\right) = \vec{\nabla}\underbrace{\left(\vec{\nabla}\cdot\vec{B}\right)}_{=0,\ \text{por (3)}} - \nabla^2\vec{B}$

Y queda: $\nabla^2\vec{B} = \mu\sigma\frac{\partial\vec{B}}{\partial t} + \mu\varepsilon\frac{\partial^2}{\partial t^2}\left(\vec{B}\right)$ **(*)**

$$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} \quad (1)$$

$$\vec{\nabla}\cdot\vec{B} = 0 \quad (3)$$

$$\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\varepsilon} \quad (2)$$

$$\vec{\nabla}\times\vec{B} = \mu\sigma\vec{E} + \mu\varepsilon\frac{\partial\vec{E}}{\partial t} \quad (4)$$

Tomo el rotacional de (1): $\vec{\nabla}\times\left(\vec{\nabla}\times\vec{E}\right) = -\frac{\partial}{\partial t}\left(\vec{\nabla}\times\vec{B}\right)$

Sustituyo (4) y obtengo: $\vec{\nabla}\times\left(\vec{\nabla}\times\vec{E}\right) = -\mu\sigma\frac{\partial\vec{E}}{\partial t} - \mu\varepsilon\frac{\partial^2}{\partial t^2}\left(\vec{E}\right)$

Desarrollo el 1er miembr $\vec{\nabla}\times\left(\vec{\nabla}\times\vec{E}\right) = \vec{\nabla}\left(\underbrace{\vec{\nabla}\cdot\vec{E}}_{=\rho/\varepsilon}\right) - \nabla^2\vec{E}$

$=\rho/\varepsilon$, por (2)

Y queda: $\nabla^2\vec{E} - \vec{\nabla}\left(\rho/\varepsilon\right) = \mu\sigma\frac{\partial\vec{E}}{\partial t} + \mu\varepsilon\frac{\partial^2}{\partial t^2}\left(\vec{E}\right)$ (* *)

$$\nabla^2 \vec{B} = \mu\sigma \frac{\partial \vec{B}}{\partial t} + \mu\varepsilon \frac{\partial^2}{\partial t^2}\left(\vec{B}\right) \quad [\,*\,]$$

$$\nabla^2 \vec{E} - \vec{\nabla}\left(\rho/\varepsilon\right) = \mu\sigma \frac{\partial \vec{E}}{\partial t} + \mu\varepsilon \frac{\partial^2}{\partial t^2}\left(\vec{E}\right) \quad [\,*\,*\,]$$

Medio no cargado:

$$\rho = 0$$

Medio no conductor:

$$\sigma = 0$$

$$\nabla^2 \vec{B} = \mu\varepsilon \frac{\partial^2 \vec{B}}{\partial t^2}$$

$$\nabla^2 \vec{E} = \mu\varepsilon \frac{\partial^2 \vec{E}}{\partial t^2}$$

ECUACIONES DE ONDA PARA LOS CAMPOS MAGNÉTICO Y ELÉCTRICO

$\vec{E}$ y $\vec{B}$ son campos acoplados. Pueden propagarse en el espacio a una cierta velocidad, oscilando en el tiempo y gobernados por sus ec de onda

Las alteraciones del campo e.m. no quedan localizadas sino que se propagan con una velocidad. "LA LUZ ES UNA SOLUCIÓN PROPAGANTE DE LAS ECUACIONES DE MAXWELL"

- En el vacío $\chi_E=1$; $\chi_M=1$: $$\nabla^2 \vec{B} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2} \qquad \nabla^2 \vec{E} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$
- Velocidad de propagación en el vacío: $$\frac{1}{v^2} \leftrightarrow \mu_0 \varepsilon_0 \qquad c = \sqrt{1/\mu_0 \varepsilon_0}$$
- Velocidad en otro medio: $$v = \sqrt{1/\mu\varepsilon}$$
- Índice de refracción de un medio: $$n = \frac{c}{v} = \sqrt{\mu\varepsilon/\mu_0\varepsilon_0} = \sqrt{\chi_M \chi_E}$$ si $\chi_M=1$: $n = \sqrt{\chi_E}$
- También pueden obtenerse para los campos $\vec{H}$ y $\vec{D}$: $$\nabla^2 \vec{H} = \mu\varepsilon \frac{\partial^2 \vec{H}}{\partial t^2} \qquad \nabla^2 \vec{D} = \mu\varepsilon \frac{\partial^2 \vec{D}}{\partial t^2}$$

ECUACIÓN EIKONAL: λ es muy pequeño (Óptica Geométrica)

En un medio de índice n, *libre de cargas y corrientes*, una perturbación electromagnética procedente de una fuente lejana (distancia >>λ) se puede escribir como

$\vec{E}_0 = \vec{e}(\vec{r})e^{ik_0 S(\vec{r})}$ (similar para $\vec{H}_0$); $S(\vec{r})$ significa "***camino óptico***"

De las **ecuaciones de Maxwell** con $k_0 \to \infty$ $(\lambda_0 \to 0)$

$$\vec{\nabla} S \times \vec{h} + \varepsilon \vec{e} = 0$$

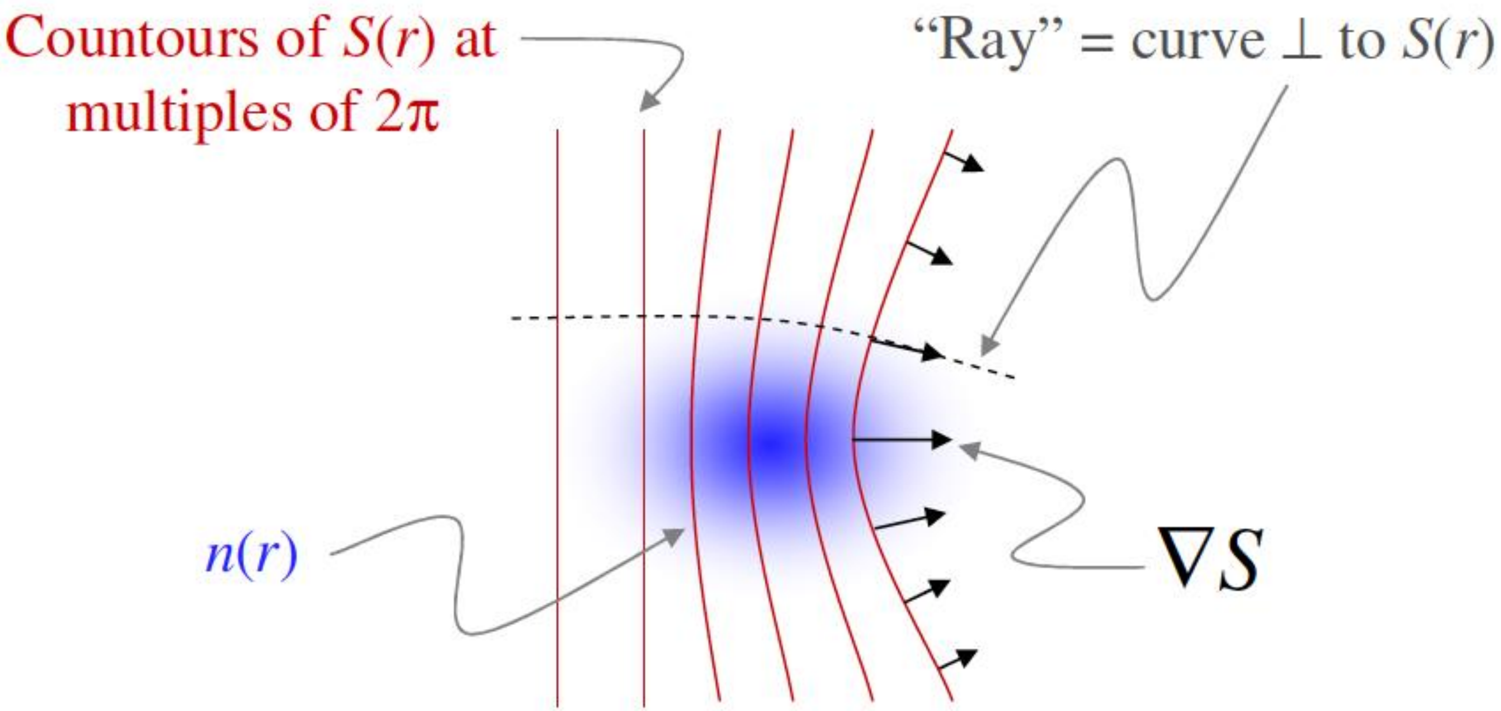
$$\vec{\nabla} S \times \vec{e} - \mu \vec{h} = 0$$ (no perder de vista que $\vec{e}\vec{\nabla} S = 0$ y $\vec{h}\vec{\nabla} S = 0$)

Y de aquí

$$\frac{1}{\mu}\left[\underbrace{(\vec{e}\vec{\nabla} S)}_{=0;\ \vec{\nabla}\vec{E}=0}\vec{\nabla} S - \vec{e}\left(\vec{\nabla} S\right)^2\right] + \varepsilon \vec{e} = 0 \Rightarrow \left(\vec{\nabla} S\right)^2 = n^2 \text{ ó } \vec{\nabla} S = n\vec{s}\ ;\ n = 1/\sqrt{\mu\varepsilon}\ ;\ \vec{s} \propto Poynting\ vector$$

CONCLUSIÓN: $S(\vec{r})$ es el camino óptico que siguen los RAYOS ($\vec{\nabla} S \propto \vec{s}$) perpendicularmente a $S(\vec{r})$ = cte (frente de onda), en los que $\vec{e}$, $\vec{h}$ y $\vec{s}$ forman un triedro tri-rectángulo

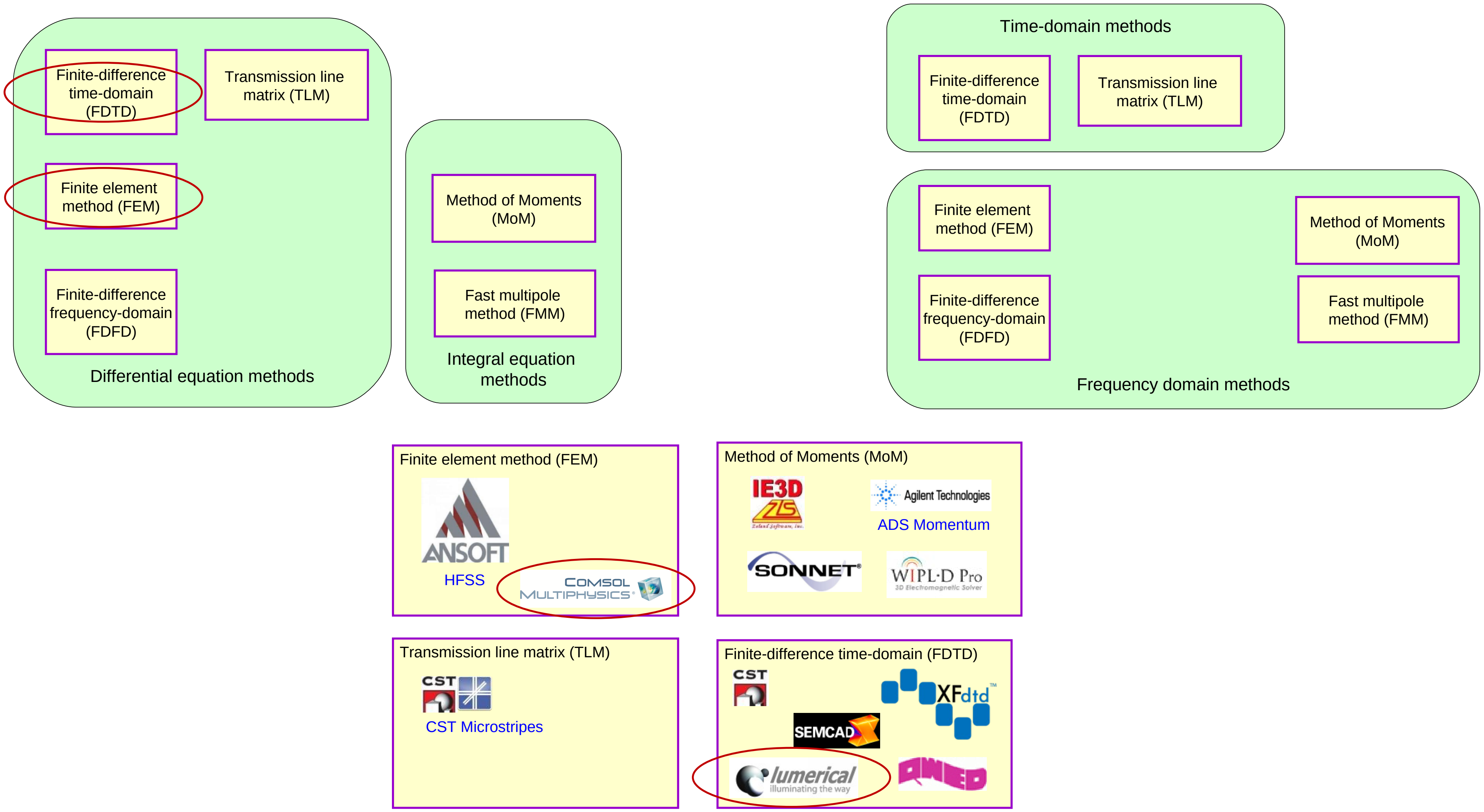
“Geometrical optics is either very simple, or else it is very complicated”
Richard P. Feynman



$$S(r) \qquad \text{Optical path length} = \int_A^B n(\vec{r})ds \qquad [\text{m}]$$

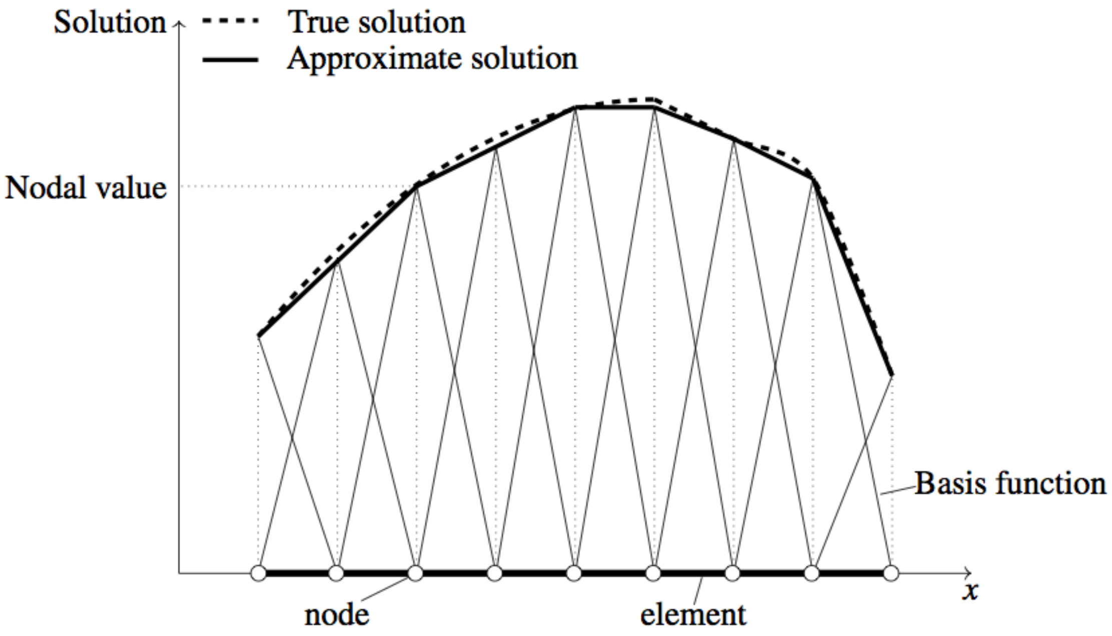
Robert McLeod (https://www.colorado.edu/ecee/robert-mcleod)

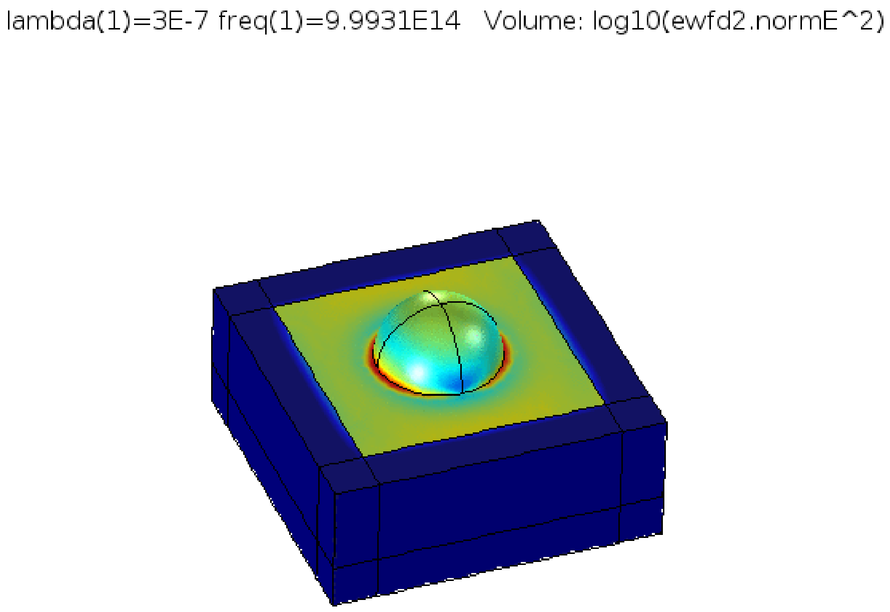
FORMAS NUMÉRICAS DE RESOLVER LAS ECUACIONES DE MAXWELL



FEM

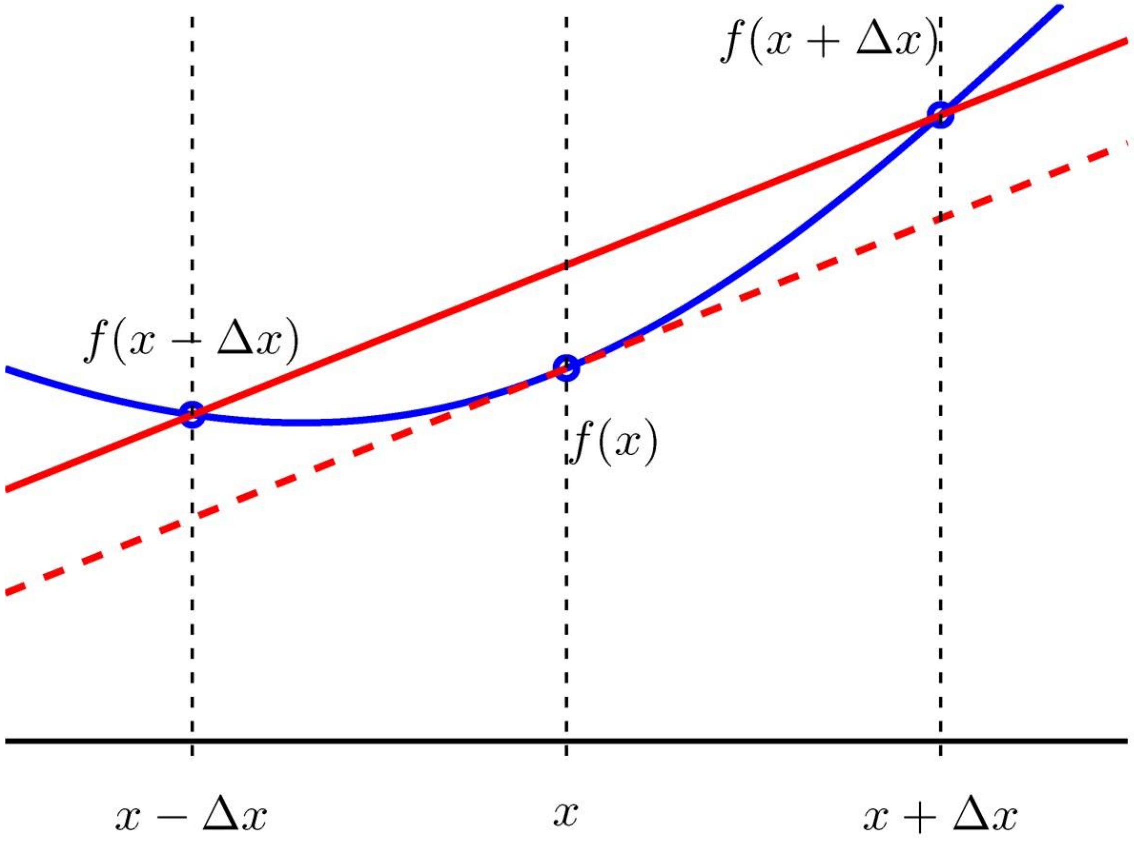
FEM discretize the domain of interest, where the PDE is defined → approximate solution of the PDE by a linear combination of basis functions defined within each subdomain.





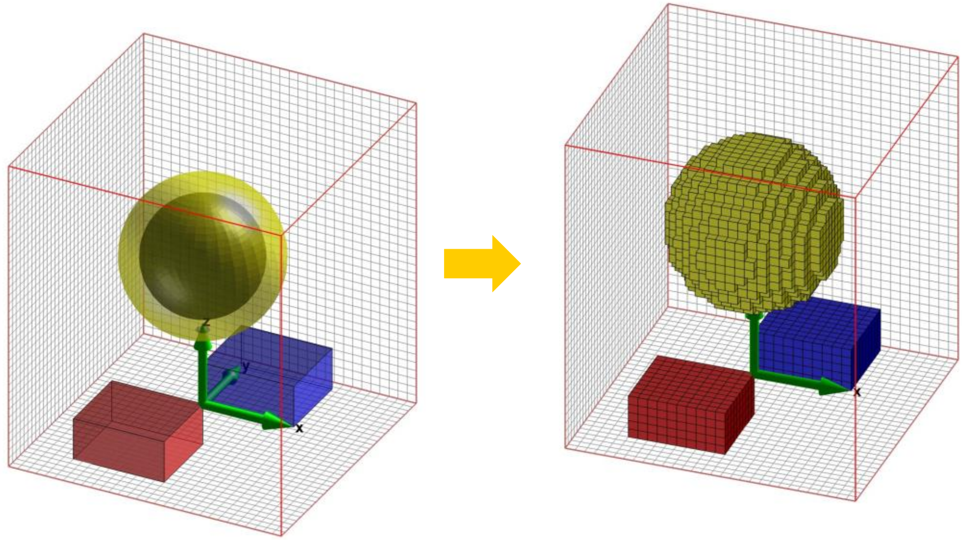
FDTD

- Represent the derivatives in Maxwell’s curl equations by finite differences
- We use the second-order accurate central difference formula

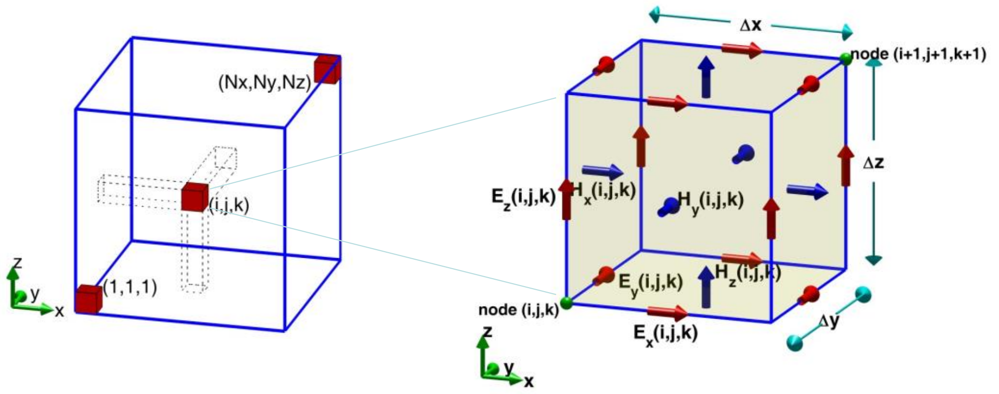


$$\frac{df(x)}{dx} = f'(x) \approx \frac{f(x+\Delta x) - f(x-\Delta x)}{2\Delta x}$$

FDTD: La celda de Yee

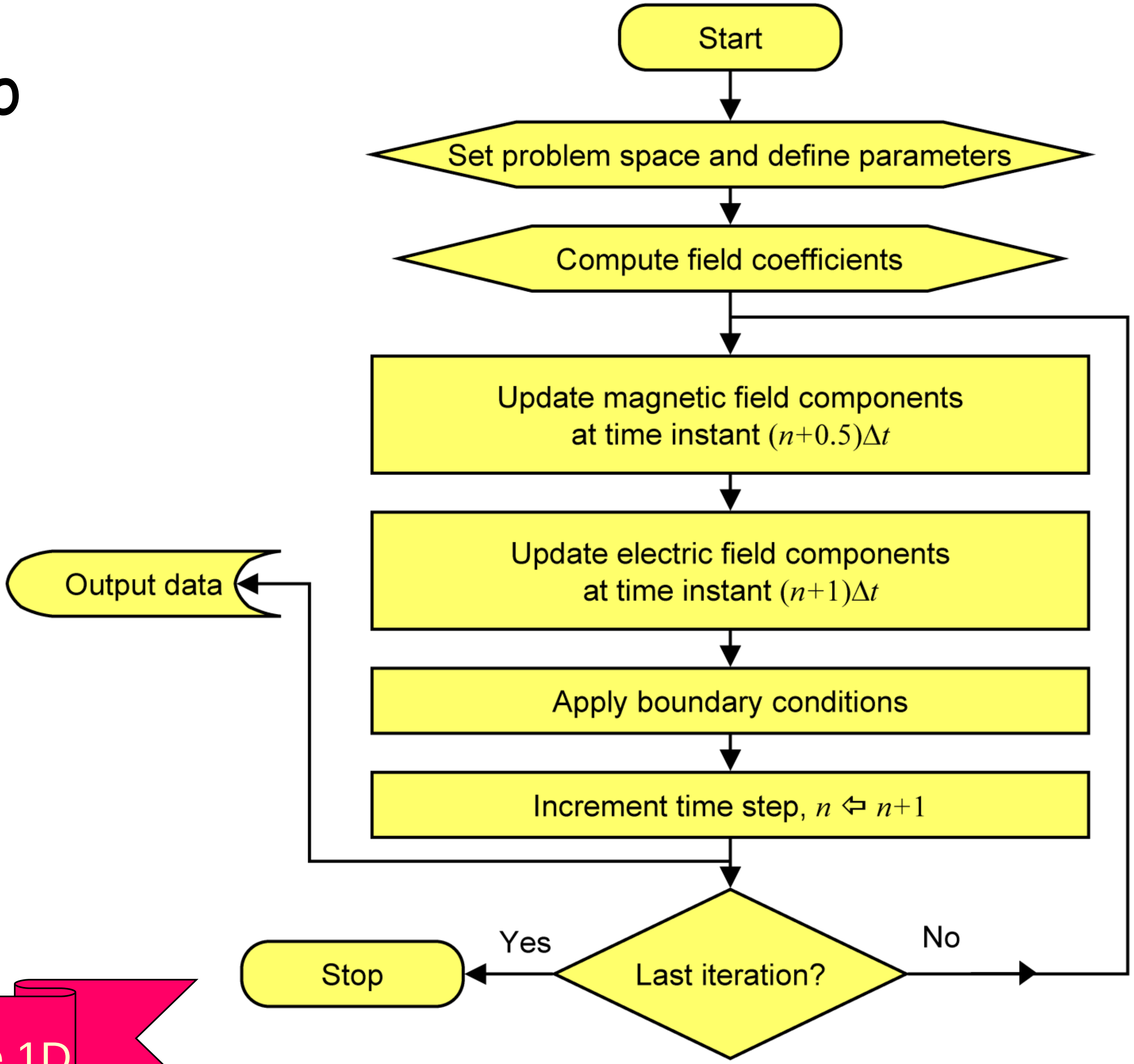


- The FDTD (Finite Difference Time Domain) algorithm was first established by Yee as a three dimensional solution of Maxwell's curl equations.



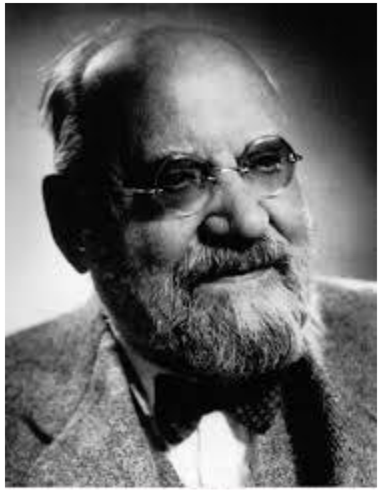
K. Yee, IEEE Transactions on Antennas and Propagation, May 1966.

El algoritmo



Exercise 1D

EJEMPLO DE SOLUCION ANALITICA: MIE THEORY



Perhaps, one of the best well stablished electromagnetic theories is

Gustav Adolf Feodor Wilhelm Ludwig Mie (29 September 1869 – 13 February 1957)

LIGHT SCATTERING BY SMALL PARTICLES

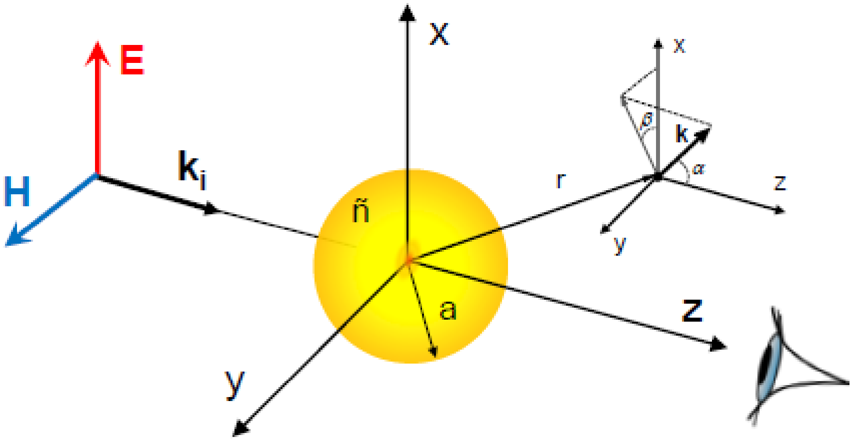
✓ It contains so much Physics that I think that there are still many things **sleeping** there.

✓ Two examples that **woke it up** recently because we needed them

PLASMONICS

HIGH REFRACTIVE INDEX PARTICLES

Mie Theory



$$\mathbf{E}_s = \sum_{n=1}^{\infty} E_n \left(i a_n \mathbf{N}_{e1n}^{(3)} - b_n \mathbf{M}_{o1n}^{(3)} \right),$$

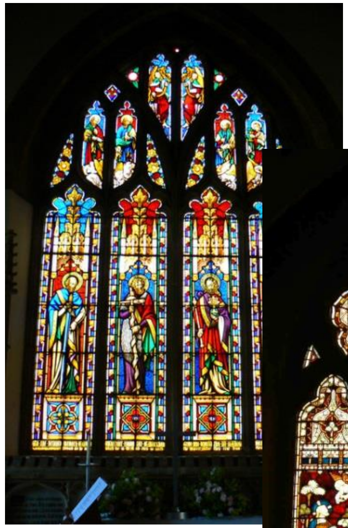
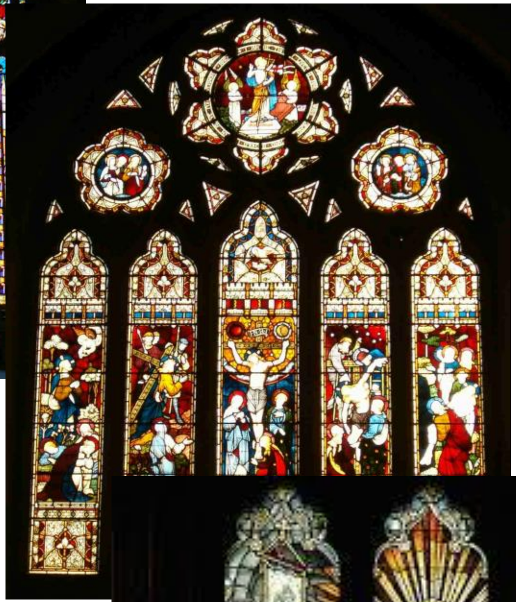
$$\mathbf{H}_s = \frac{k}{\omega\mu} \sum_{n=1}^{\infty} E_n \left(i b_n \mathbf{N}_{o1n}^{(3)} + a_n \mathbf{M}_{e1n}^{(3)} \right),$$

$$a_n = \frac{\mu m^2 j_n(mx)[x j_n(x)]' - \mu_1 j_n(x)[mx j_n(mx)]'}{\mu m^2 j_n(mx)\left[x h_n^{(1)}(x)\right]' - \mu_1 h_n^{(1)}(x)[mx j_n(mx)]'},$$

$$b_n = \frac{\mu_1 j_n(mx)[x j_n(x)]' - \mu j_n(x)[mx j_n(mx)]'}{\mu_1 j_n(mx)\left[x h_n^{(1)}(x)\right]' - \mu h_n^{(1)}(x)[mx j_n(mx)]'},$$

a_n, b_n: optical/size properties **N, M**: geometrical properties

Los ancestros de la nanotecnología

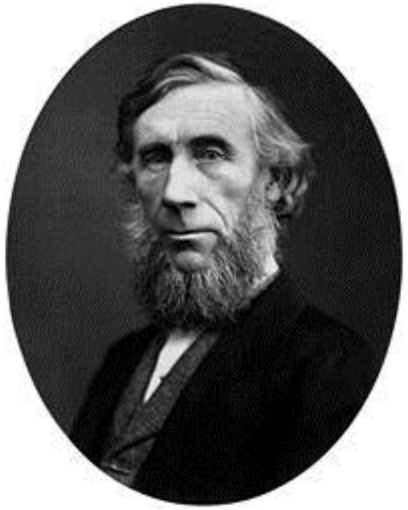




Vidrieras

La copa de Lycurgus

(glass; British Museum; 4th century A. D.)

Dos grandes de lo NANO



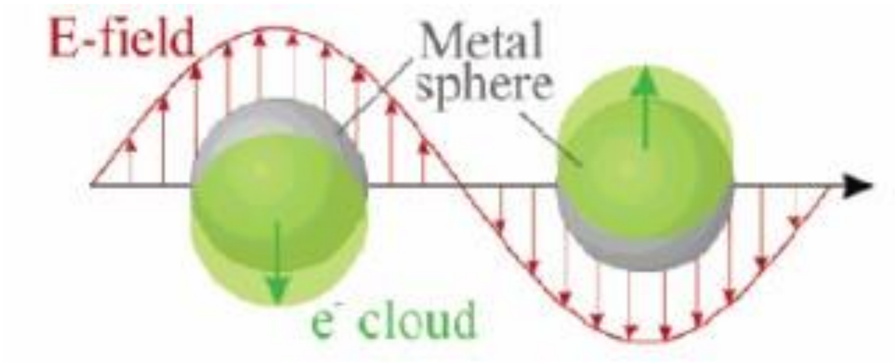
John Tyndall
(1820-1893)

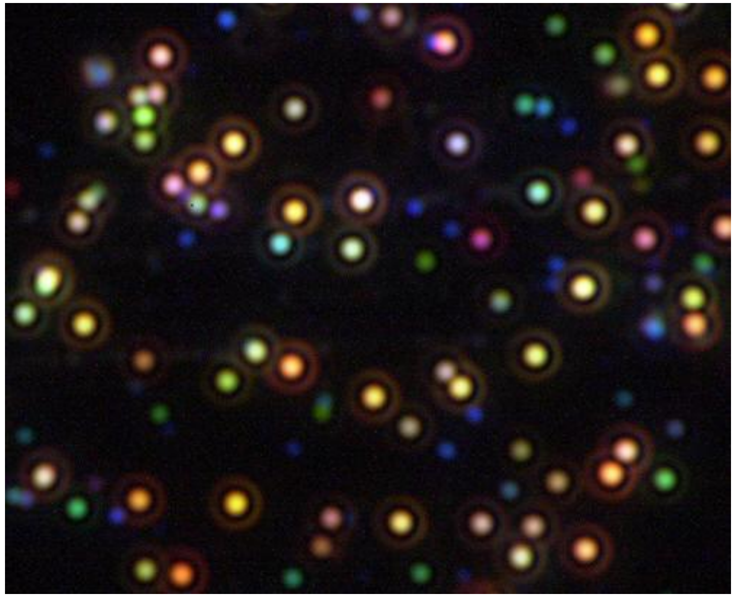
Michael Faraday
(1791-1867)

EFECTO TYNDALL → COLOR AZUL DE OJOS

DIFUSIÓN RAYLEIGH, COLOR AZUL DEL CIELO

Metales
Plasmones localizados: nanopartículas



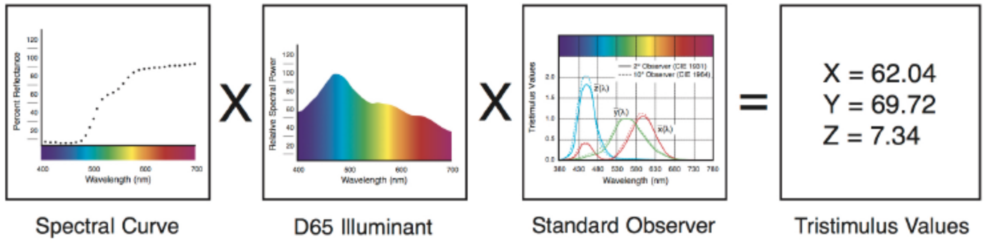


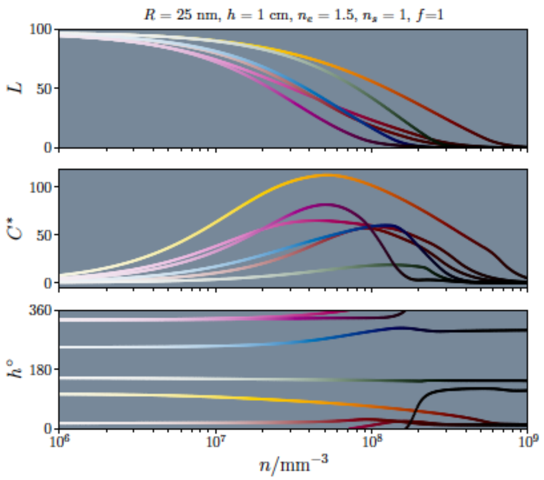
- Forma
- Tamaño
- Propiedades ópticas (Ag,Au)

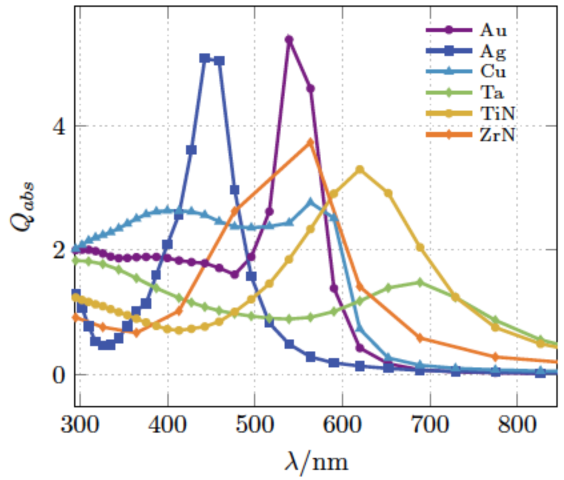
Sintonía en el visible, IR

Un ejemplo "home-made"

ASSUMING A STANDARD ILLUMINANT AND OBSERVER

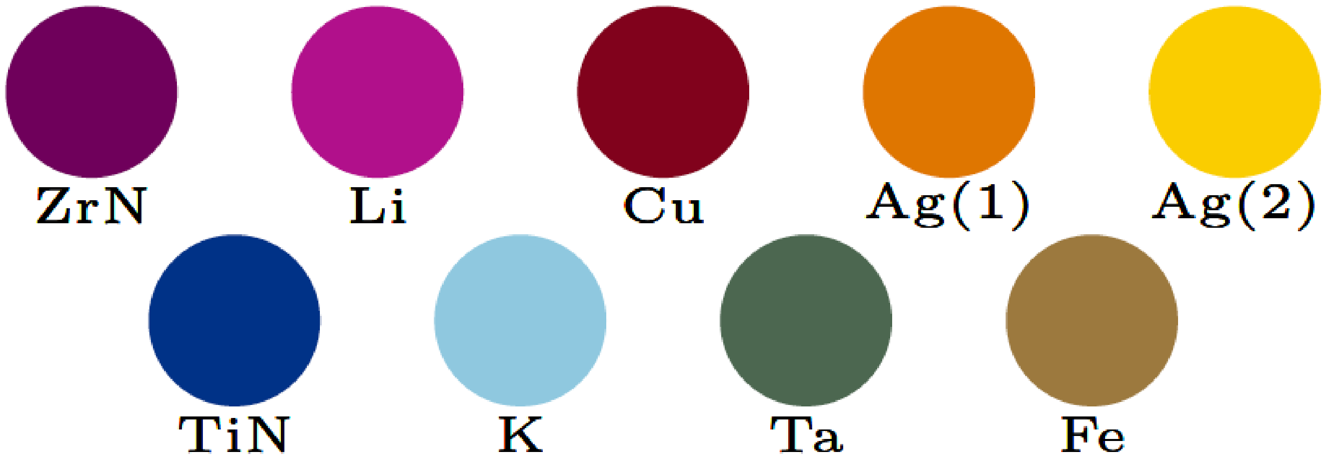






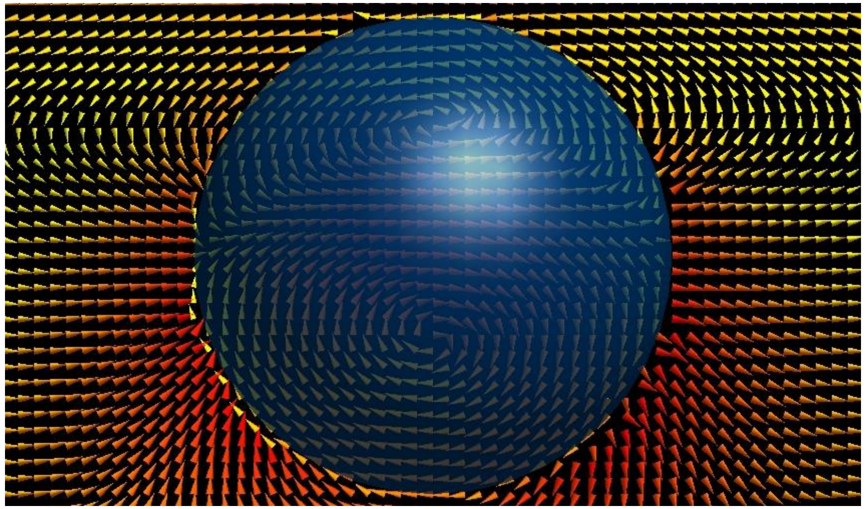
COLOR COORDINATES CONVERSION

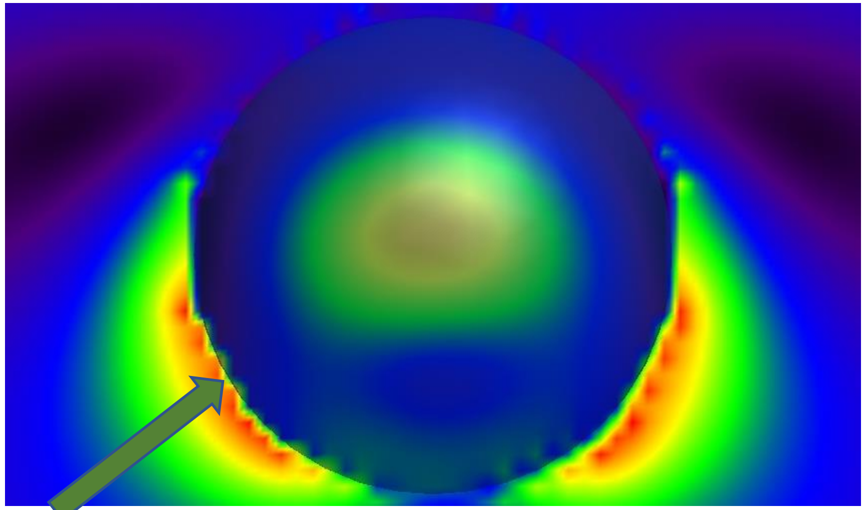
$X, Y, Z \rightarrow L^*, a^*, b^* \rightarrow L^*, C^*, h°$



Rodrigo Alcararaz
Elena de la Hoz

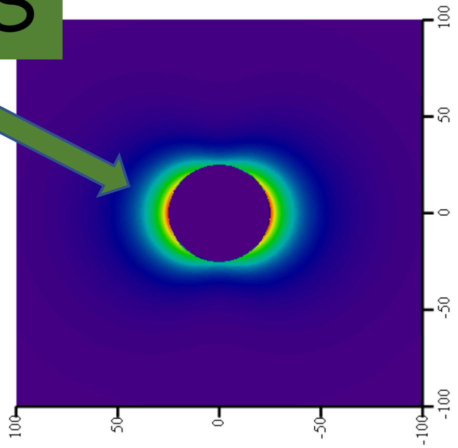
$$a_1\,(\varepsilon_{eff} = -2) \rightarrow \quad \alpha_E^{res} = \frac{6\pi i a_1}{k^3}$$





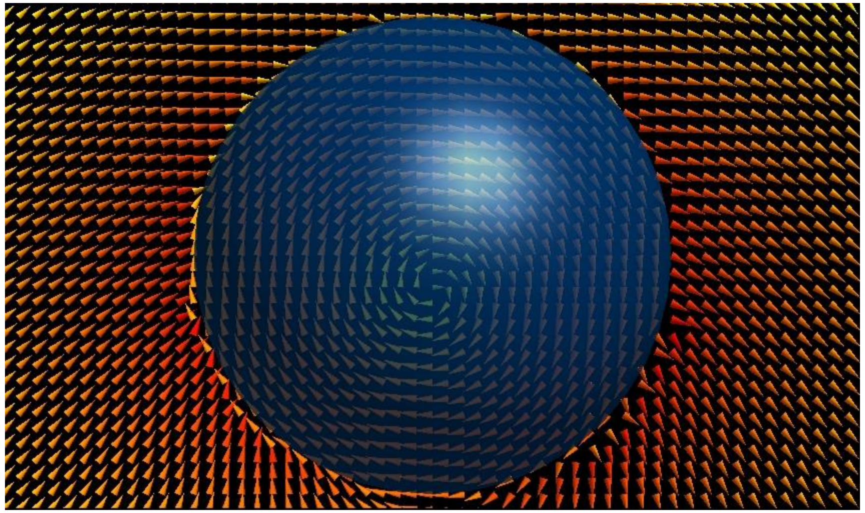
HOT-SPOTS

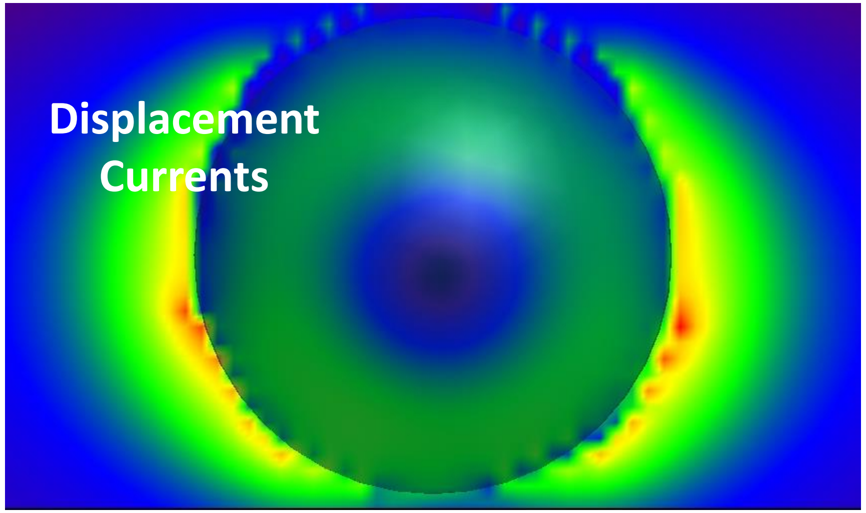
~ Plasmon in metallic nanoparticles



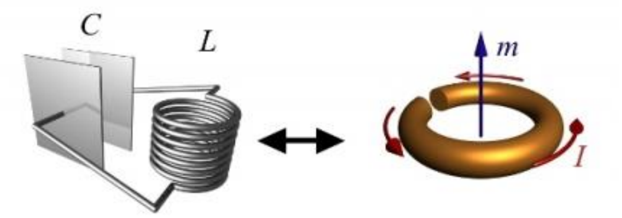
Ag,
R=25nm,
λ=355nm,
ε= -2+0.3i

$$b_1\,(\mu_{eff} = -2) \rightarrow \quad \alpha_M^{res} = \frac{6\pi i b_1}{k^3}$$

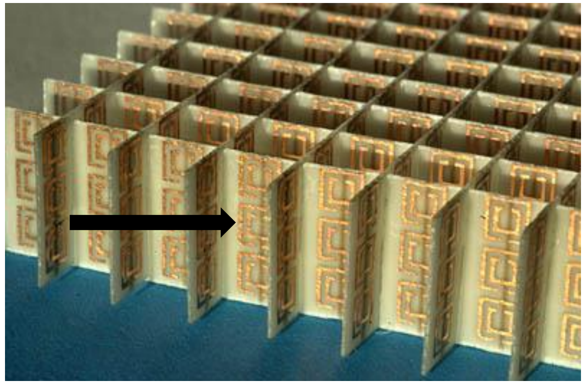




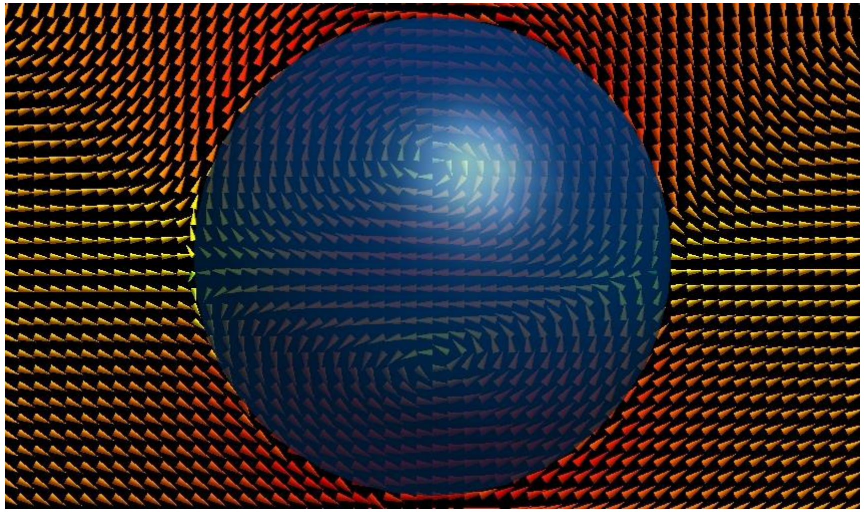
Split Ring
Resonators

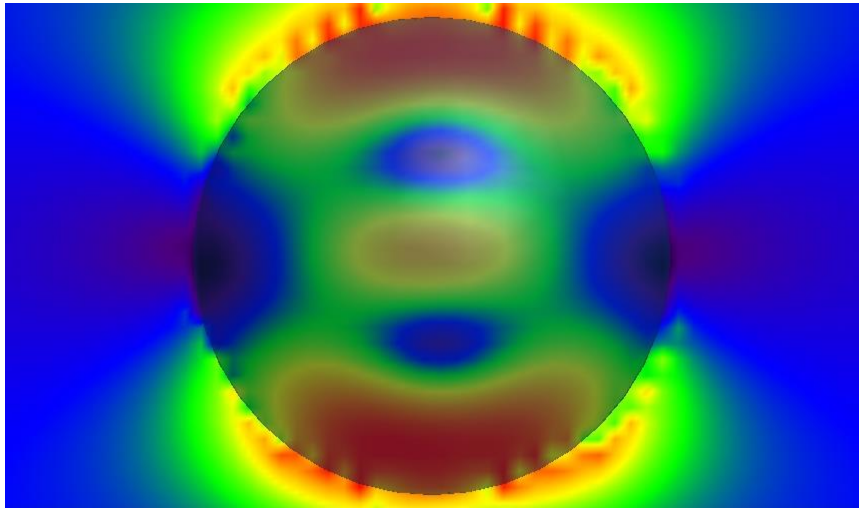


Prof. Wegener webpage-KIT (Germany)

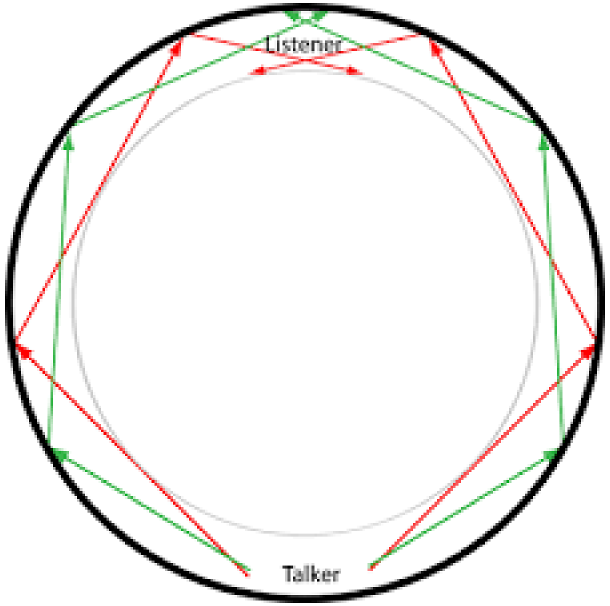


$b_2\,(\mu_{eff} = -1.5)$





WHISPERING GALLERY MODES (RESONANCIAS)

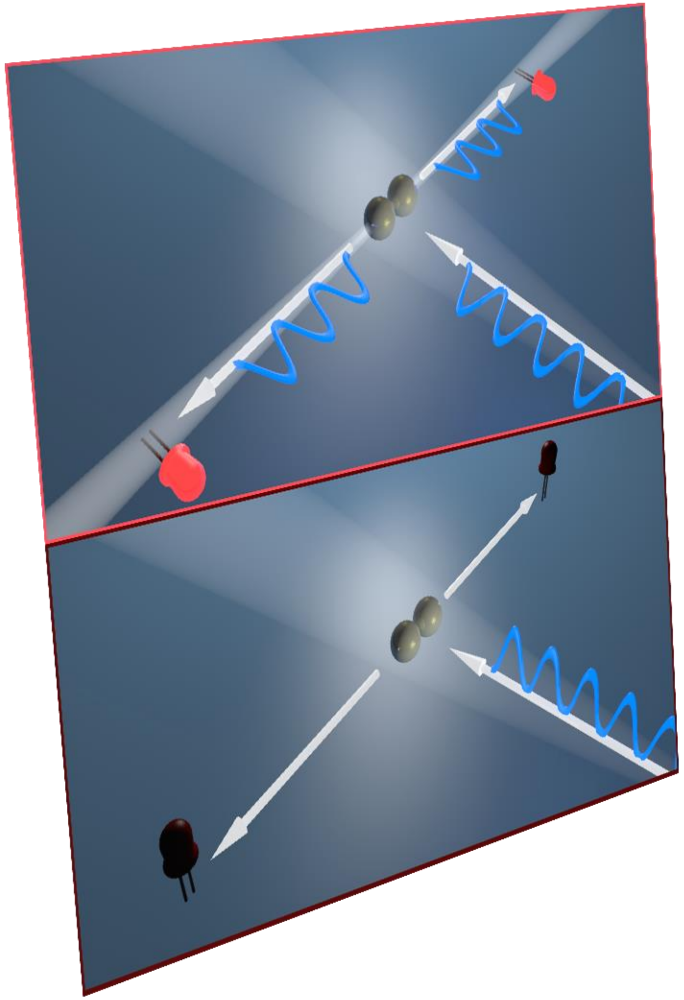


Catedral de San Pablo en Londres (S. Paul’s Cathedral)

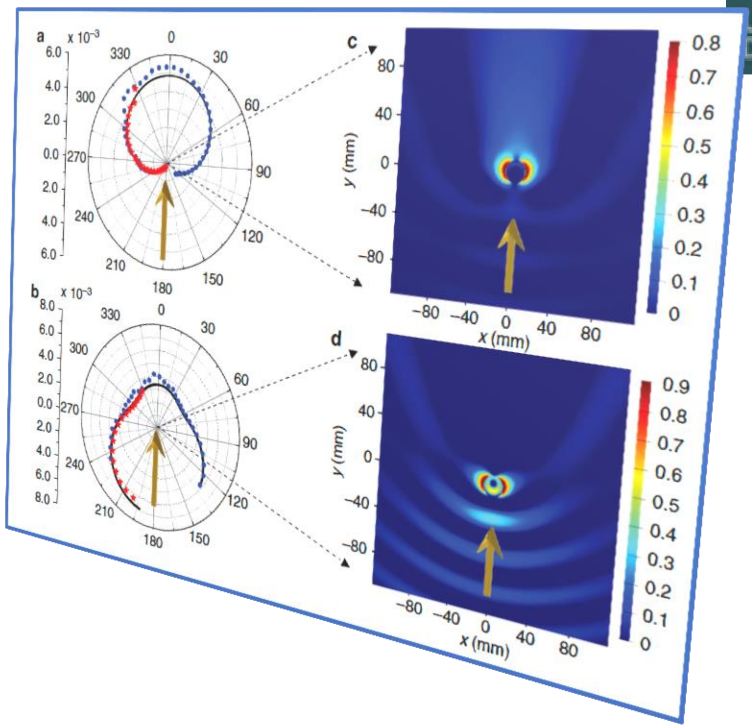
Appl Phys B (2010) 100: 847–850

POR <u>REESCALADO</u> Y <u>REDUCIENDO LA LONGITUD DE ONDA</u> USANDO <u>MATERIALES DENSOS</u> (alto indice de refracción) COMO EL <u>SILICIO</u> EN EL *VIS/NIR*, SE PUEDE CONSEGUIR EL <u>MISMO EFECTO CON LUZ</u>

Partículas de Si de 300nm



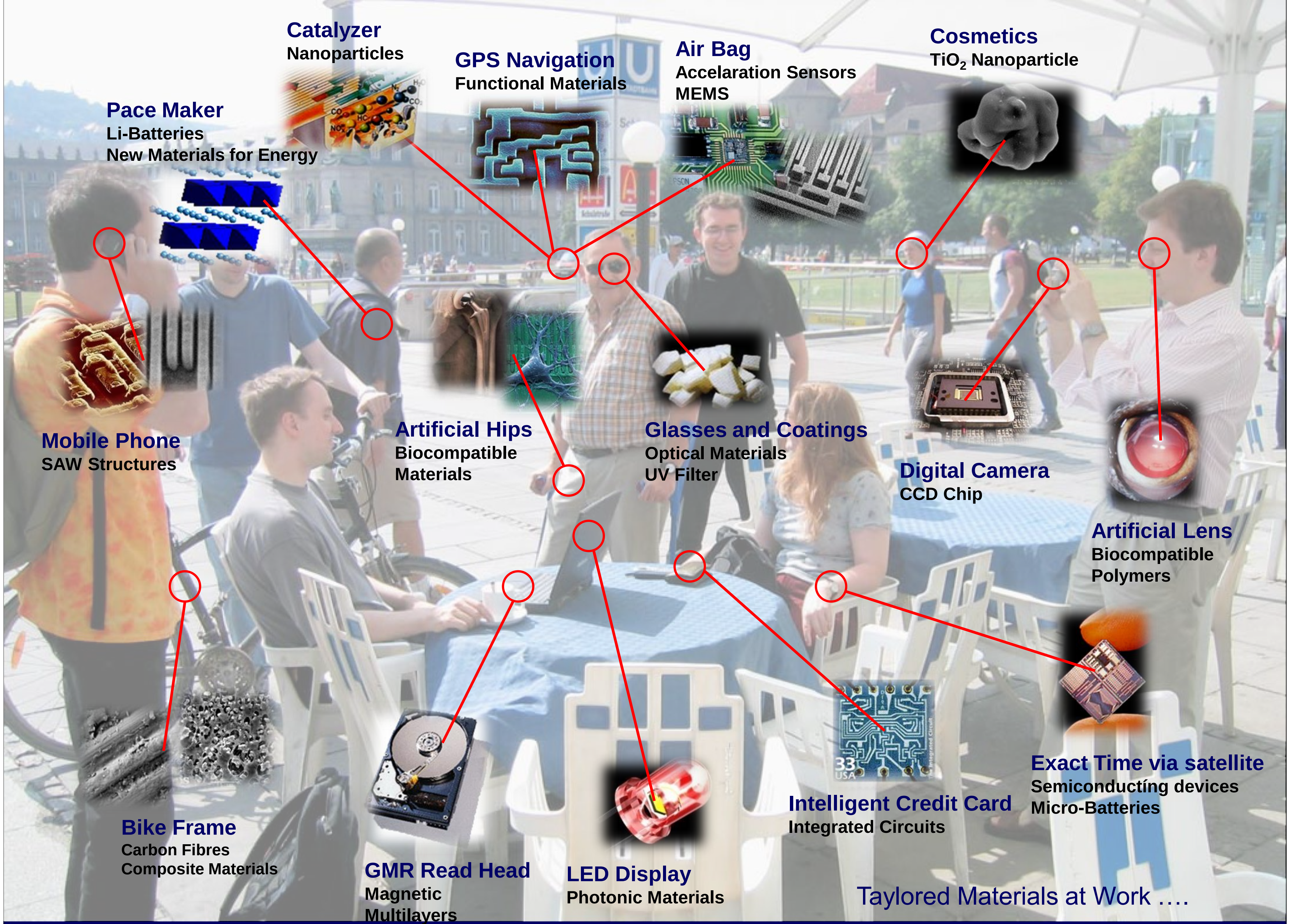
NANO-ANTENAS



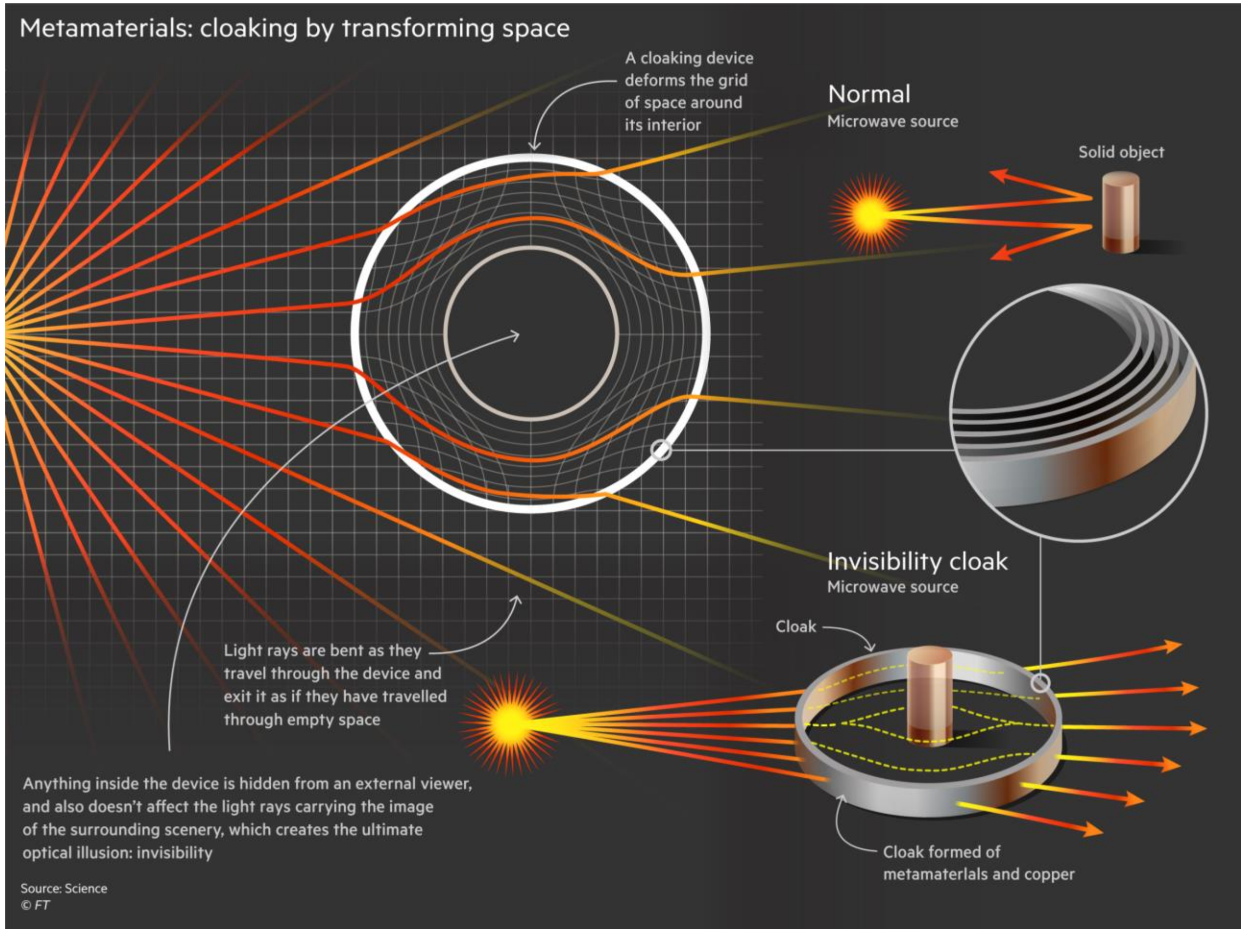
INTERRUPTOR ÓPTICO→
COMPUTACIÓN ÓPTICA

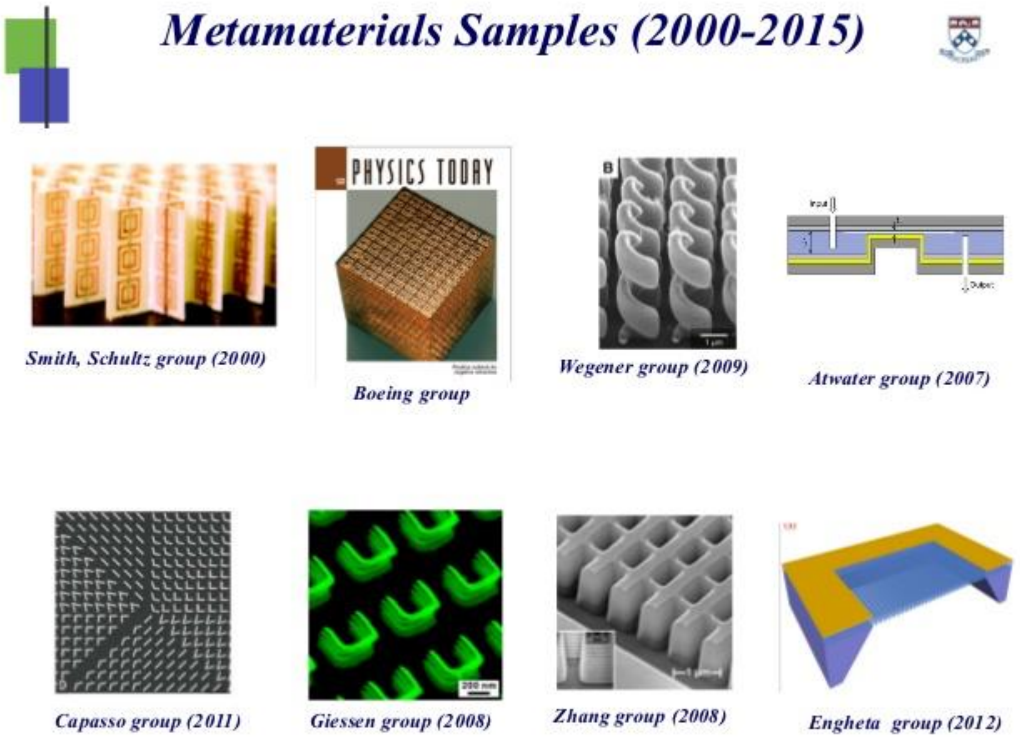
On a typical day somewhere in the world….

UNIVERSIDAD DE CANTABRIA


Catalyzer
Nanoparticles
GPS Navigation
Functional Materials
Air Bag
Accelaration Sensors
MEMS
Cosmetics
TiO_2 Nanoparticle
Pace Maker
Li-Batteries
New Materials for Energy
Mobile Phone
SAW Structures
Artificial Hips
Biocompatible
Materials
Glasses and Coatings
Optical Materials
UV Filter
Digital Camera
CCD Chip
Artificial Lens
Biocompatible
Polymers
Exact Time via satellite
Semiconducting devices
Micro-Batteries
Intelligent Credit Card
Integrated Circuits
Bike Frame
Carbon Fibres
Composite Materials
GMR Read Head
Magnetic
Multilayers
LED Display
Photonic Materials
Taylored Materials at Work ….

LOS METAMATERIALES





MUCHAS GRACIAS